РОЗДІЛ 1. АКТУАЛЬНІ ПИТАННЯ ПІДВИЩЕННЯ ЯКОСТІ НАВЧАННЯ ДИСЦИПЛІН ПРИРОДНИЧО-МАТЕМАТИЧНОГО ЦИКЛУ В ШКОЛІ ТА ВИЩИХ НАВЧАЛЬНИХ ЗАКЛАДАХ РІЗНИХ РІВНІВ АКРЕДИТАЦІЇ

UDC 372.851
DOI 10.5281/zenodo.3547758

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MODERN THEMATIC PREPARATION FOR EIA IN MATHEMATICS IN UKRAINE: NUMBERS AND EXPRESSIONS, FUNCTIONS

The relevance of research on the thematic preparation for the EIA in mathematics in modern Ukrainian realities is not in doubt. Based on the experience of systematization and repetition of the school course in mathematics, we have proposed dividing the entire course of mathematics into 10 thematic semantic blocks: "Numbers and Expressions", "Functions", "Equations and Systems of Equations", "Inequalities and Systems of Inequalities", "Text Problems", "Elements of mathematical analysis", "Planimetry", "Stereometry", "Coordinates and vectors", "Elements of combinatorics and stochastics".

In the article, we propose thematic tests to the first two substantial blocks ("Numbers and Expressions" and "Functions"), as well as the answers to them. In addition, we solve the basic tasks of these tests and give methodological comments on these solutions. We are convinced that a properly organized thematic repetition of the school course in mathematics will allow teachers to excel in preparing students for independent testing in mathematics.

Key words: *IEA in mathematics, SFA in mathematics, thematic preparation, educational achievements of students, thematic tests, basic tasks, numbers and expressions, functions.*

Formulation of the problem. External Independent Assessment (EIA) is now the main instrument of assessing the quality of mathematical preparation for Ukrainian graduates. In particular, it is used for conducting the State Final Attestation (SFA) of academic achievements of senior school students, as well as as a tool for competitive selection of applicants to Ukrainian universities. Therefore, there is no doubt about the relevance and the need for research on various aspects of preparation for the EIA in mathematics.

One such aspect is the systematic and thematic repetition of the school mathematics course. Based on our many years of experience in preparing for EIA, during this repetition we divide the whole mathematics course into 10 thematic blocks: «Numbers and Expressions», «Functions», «Equations and Systems of Equations», «Inequalities and Systems of Inequalities», «Text Problems», «Elements of mathematical analysis», «Geometry on the plane», «Geometry in the Space», «Coordinates and vectors», «Elements of combinatorics and stochastics».

It is this division that allows repeated repetition of the same material throughout the preparation process for the EIA. For example, the transformations of trigonometric expressions are repeated during the study of thematic blocks 1, 2, 3, 6, 7, 8 and 9. This permits the teacher constantly, as one said, to keep the student in a tone, when he would forget something, but he can't do this, because proposed thematic training system doesn't allow it.

Analysis of current research. The problem of preparing students for EIA in mathematics is systematically considered in scientific and pedagogical publications. Constantly publish the results of their research in this area of research Valentyna Bevz, Mykhailo Burda, Hryhoriy Bilyanin, Olga Bilyanina, Olga Vashulenko, Larysa Dvoretska, Oxana Yergina, Oleksandr Ister, Vadym Karpik, Arkadiy Merzlyak, Yevgen Nelin, Victor Repeta, Oleksiy Tomaschuk, Mykhailo Yakir and others. During the last 15 years, our author's team has been constantly working to provide methodological support for the process of preparation for the EIA in mathematics. The theory and methodology of evaluating the academic achievement of senior school students in Ukraine is described in the monograph [1]. For the repetition and systematization of the school mathematics course, we use the methodological set of manuals [2] and [3]. Previously, we have considered certain aspects of thematic preparation for independent testing, but since then the contingent of EIA participants has changed significantly, as well as the methodological views of our author's team on this problem are also evolved.

The purpose of the article. The purpose of this article is to provide methodological advice to teachers and tutors regarding the thematic preparation of graduates to EIA in mathematics. In particular, we present in this article two thematic tests related to the topics «Numbers and Expressions» and «Functions», and also provide a solution of the some basic tasks of these tests with methodical comments for them.

Research methods. In order to achieve this goal we use in this paper some theoretical methods, such as an analysis of methodological literature on the research subject. Also we apply some empirical methods: observation of the training process of the students during their studying on training courses for the EIA in mathematics and analysis of the results of their achievements. The research also used a set of methods of scientific cognition: a comparative analysis to find out different views on the problem and determine the direction of research; systematization and generalization for the formulation of conclusions and recommendations; generalization of author's pedagogical experience and observations.

Presenting main material. We believe that in preparing for the EIA, it is advisable to refrain from a variety of problem forms in the repetition and systematization of the material of each topic, limiting only to open-ended tasks with full explanation, as they are the most effective for teaching mathematics and feedback. However, after completing each of the 10 thematic blocks, it is natural to carry out a diagnostic thematic test in which to use all forms of test tasks inherent in the EIT math test.

Thematic test «Numbers and Expressions».

Tasks 1-7 have five answer choices, only one of which is correct. Choose the correct answer, in your opinion.

1. Calculate: $(0,2-1):0,04$.

A	B	C	D	E
– 0,02	0,02	– 0,2	20	– 20

2. $\frac{1}{2}+\frac{1}{3}\cdot\frac{1}{4}=$

A	B	C	D	E
$\frac{5}{24}$	$\frac{7}{12}$	$\frac{2}{14}$	$\frac{1}{24}$	$\frac{3}{9}$

3. $\frac{x^2-5x}{5x}=$

A	B	C	D	E
x^2	x^2-1	$\frac{x-5}{5}$	$\frac{x-5}{x}$	$\frac{x}{5}$

4. Specify the expression whose numeric value is *the smallest*.

A	B	C	D	E
$2\cdot\sqrt{3}$	$\sqrt{34}:\sqrt{2}$	$\sqrt{5}\cdot\sqrt{3}$	$\sqrt{39}:\sqrt{3}$	$3\cdot\sqrt{2}$

5. $\left(x^2\right)^8:x^4=$

A	B	C	D	E
x^{20}	$x^{\frac{5}{2}}$	x^6	x^{12}	x^4

6. Simplify the expression: $\left(\cos^2\alpha - \sin^2(-\alpha)\right)\cdot \mathrm{tg}(2\alpha)$.

A	B	C	D	E
$\frac{\cos^2 2\alpha}{\sin 2\alpha}$	$\frac{\sin^2 2\alpha}{\cos 2\alpha}$	$\sin 2\alpha$	$\mathrm{tg}(2\alpha)$	$\cos 2\alpha$

7. Simplify the expression $3a+4+\sqrt{(3a+1)^2}$, if $3a+1<0$.

A	B	C	D	E
3	5	$6a+3$	$6a+5$	$9a^2+9a+5$

In Task 8 for each of the three rows of data marked with numbers, select the one correct, in your opinion, variant marked with a letter.

8. Match the expression (**1** – **3**) to its numeric value (**A** – **E**).

Expression	*Numeric value of expression*
1 $\left(\frac{2}{\sqrt{6}}\right)^2$	**A** $\frac{1}{3}$
2 $6\sin 15^\circ \cos 15^\circ$	**B** $\frac{3\sqrt{3}}{2}$
3 $\log_6 2 - \log_6 \frac{1}{3}$	**C** $\frac{2}{3}$
	D 1
	E $\frac{3}{2}$

Solve Tasks 9-11. Record the numeric answers you received *in decimal or integer*.

9. We know that $a^2 = 9b^2 + 125\sqrt{2}$. Find the expression values:

$$1)\ \frac{(a+3b)(a-3b)}{\sqrt{2}};\ 2)\ \log_5(a+3b)+\log_5(a-3b)-0{,}5\log_5 2.$$

10. Find p, if $\sqrt{\frac{2\sqrt[4]{2}}{\sqrt{2}}} = 2^p$.

11. Calculate: $9^{\frac{-2}{\lg 9}} + \log_{121}\sqrt{11}$.

Solve Task 12. Write down sequential logical actions and explanations of all stages of task solving, make reference to the mathematical facts from which one or another statement follows. If necessary, illustrate the task solving with drawings, graphs, etc.

12. We know that $\sin x + \cos x = -1{,}2$. Find the expression values:

$$1)\ \sin 2x;\ 2)\ \sin^3 x + \cos^3 x.$$

Answers to test «Numbers and expressions»

1	2	3	4	5	6	7	8	9	10	11
E	B	C	A	D	C	A	1 – C; 2 – E; 3 – D	1) 125; 2) 3	0,375	0,26

12. 1) 0,44; 2) $-0{,}936$

Solutions and comments to tasks of test «Numbers and expressions».

Task 7 (term of the task see above). *Solution.* Whereas $\sqrt{a^2} = |a|$, then $3a+4+\sqrt{(3a+1)^2} = 3a+4+|3a+1|$. By the definition of absolute value, if $3a+1<0$, then $|3a+1| = -3a-1$, therefore, $3a+4+\sqrt{(3a+1)^2} = 3a+4-3a-1 = 3$ and the correct answer is **A**.

Comment. This task is aimed primarily at testing the knowledge and understanding of the formula $\sqrt{a^2}=|a|$, because many students mistakenly believe that $\sqrt{a^2}=a$ and they choose the distractor **D**. In addition, understanding the notion of absolute value is traditionally difficult for students. This is especially true of the procedure for opening the sign of absolute value in letter expressions. If the student incorrectly opens the sign of absolute value, then he will again select distractor **D**, and if there is a technical error, then he will choose distractor **B** or **C**. Finally, distractor **E** can be obtained by ignoring the root sign in the expression.

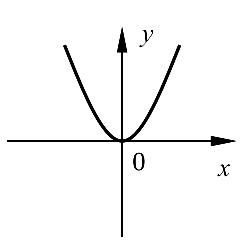
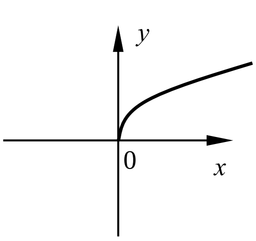
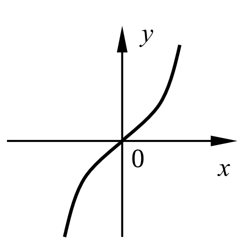
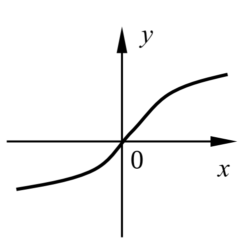
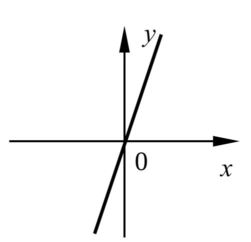
Task 10 (term of the task see above). *Solution.* Let's transform an expression using the properties of the roots: $\sqrt{\frac{2\sqrt[4]{2}}{\sqrt{2}}}=\sqrt{\frac{\sqrt[4]{2^4\cdot 2}}{\sqrt[4]{2^2}}}=\sqrt{\sqrt[4]{\frac{2^5}{2^2}}}=\sqrt[8]{2^3}$. By definition of degree with fractional index: $\sqrt[8]{2^3}=2^{\frac{3}{8}}$ and $p=\mathbf{0{,}375}$.

Comment. This task, first of all, checks whether the student knows the definition of degree with fractional index, and therefore it permits an alternative solution without using the properties of the roots: $\sqrt{\frac{2\sqrt[4]{2}}{\sqrt{2}}}=\left(\frac{2^1\cdot 2^{\frac{1}{4}}}{2^{\frac{1}{2}}}\right)^{\frac{1}{2}}=\left(2^{1+\frac{1}{4}-\frac{1}{2}}\right)^{\frac{1}{2}}=\left(2^{\frac{3}{4}}\right)^{\frac{1}{2}}=2^{\frac{3}{8}}$. In addition, when preparing to solve short-answer tasks, students should pay attention to converting important fractions to decimal: $\frac{1}{2}=0,5$; $\frac{1}{4}=0,25$; $\frac{3}{4}=0,75$; $\frac{1}{8}=0,125$; $\frac{3}{8}=0,375$; $\frac{5}{8}=0,625$; $\frac{7}{8}=0,875$. If a student during solving a short-answer task gets a fraction that cannot be written in decimals ($\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{7}$ etc), then he need to check the solution, because an error was made. The same should be done if irrationality appears in the answer, e.g. $\sqrt{2}$, $\sqrt{5}$, $\sqrt[3]{9}$ etc.

Thematic test «Functions».

Tasks 1-7 have five answer choices, only one of which is correct. Choose the correct answer, in your opinion.

1. Specify the function $y=x^3$ graph.

A	B	C	D	E
				

2. Specify the function whose graph goes beyond the origin.

A	B	C	D	E
$y=x-3$	$y=-\frac{3}{x}$	$y=x+3$	$y=\frac{3}{x}$	$y=\frac{x}{3}$

3. Find the function definition area $y=\sqrt{x+1}$.

A	B	C	D	E
$[-1;+\infty)$	$[0;+\infty)$	$[1;+\infty)$	$(-1;+\infty)$	$(1;+\infty)$

4. Specify the interval at which the function $y=(x-3)^2$ increases.

A	B	C	D	E
$(-\infty;3]$	$[3;+\infty)$	$[0;+\infty)$	$(-\infty;0]$	$(-\infty;+\infty)$

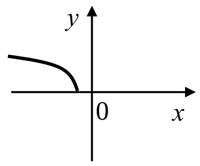
5. Specify the set of function *values* $y = \log_{0,3} x$.

A	**B**	**C**	**D**	**E**
$(-\infty;+\infty)$	$(0;1)$	$(0;+\infty)$	$(1;+\infty)$	$(0,3;+\infty)$

6. We know that a *paired* function $y = f(x)$ has only two zeros, one of which is $x = 3$. Specify the second zero of this function.

A	**B**	**C**	**D**	**E**
$-\frac{1}{3}$	0	$\frac{1}{3}$	–3	$\sqrt{3}$

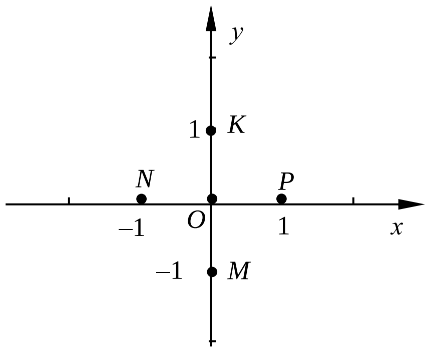
7. The figure shows a fragment of a function graph. Which of the following functions this fragment *can* belong to?



A	**B**	**C**	**D**	**E**
$y = \sqrt{-x}$	$y = 3^{-x}$	$y = \log_3(-x)$	$y = -\text{ctg}\, x$	$y = -\log_3 x$

In Task 8 for each of the three rows of data marked with numbers, select the one correct, in your opinion, variant marked with a letter.

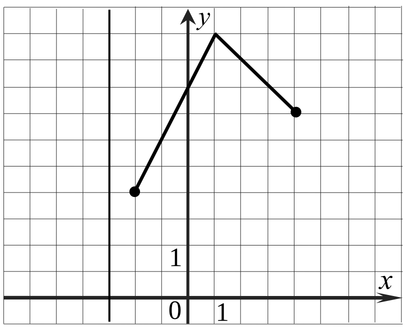
8. On the the figure you can see five points in a rectangular coordinate system. Match the function (**1** – **3**) to the point (**A** – **E**) that belongs to its graph.



Function	*Point*
1 $y = 2x$	**A** *M*
2 $y = \log_2 x$	**B** *N*
3 $y = \cos x$	**C** *O*
	D *K*
	E *P*

Solve Tasks 9-11. Record the numeric answers you received *in decimal or integer*.

9. On the graph you can see the graph of a function $y = f(x)$, defined on $[-2;4]$. Find: 1) the *smallest* value of the function $y = f(x)$ on its definition area; 2) $x_0 + y_0$, where $(x_0; y_0)$ is the common point of function graphs $y = f(x)$ and $y = 2^x$.



10. Find the *largest* value of the function $y = 5 + 2\sin x$.

11. Let $f(x) = |x - 7,3| + \log_{0,25} x$. Find $f(4)$.

Solve Task 12. Write down sequential logical actions and explanations of all stages of task solving, make reference to the mathematical facts from which one or another statement follows. If necessary, illustrate the task solving with drawings, graphs, etc.

12. Let $f(x) = \begin{cases} x^2, & x \in (-\infty;1); \\ \frac{1}{x}, & x \in [1;+\infty). \end{cases}$ 1) Find $f(-4)$; 2) plot a function $y = f(x)$ graph; 3) Find the set of values of the function $y = f(x)$; 4) Find all the values m, when the

line $y = m$ has only one common point with the graph of $y = f(x)$.

Answers to thematic test «Functions»

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
C	E	A	B	A	D	C	1 – C; 2 – E; 3 – D	1) 4; 2) 11	7	2,3

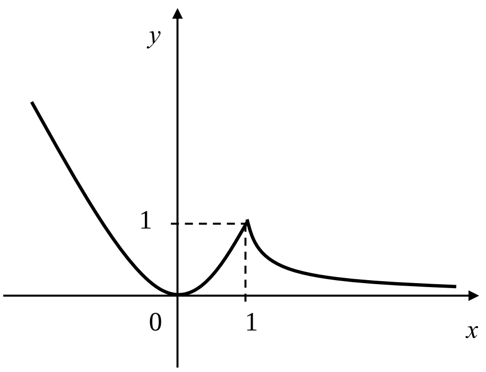
12. 1) 16; 2) see solution below; 3) $E(f) = [0;+\infty)$; 4) $m \in \{0\} \cup (1;+\infty)$.

Solutions and comments to tasks of test «Functions».

Task 9 (term of the task see above). *Solution.* 1) From the figure we can see that the smallest value of a function in its domain is **4**. 2) We sketch the function graph $y = 2^x$ n the same coordinate system as the given graph. We see that the intersection point of these graphs $M(3;8)$, therefore, $x_0 + y_0 = 3 + 8 = \mathbf{11}$.

Comment. This task tests the ability to read graphs, understanding the concept of the largest and smallest value of a function on a segment, as well as the ability to plot the graph of the exponential function $y = 2^x$. It should be noted by the students that quite often the function on this segment acquires its extreme values at the end points of this segment. When analyzing Task 9, you can also ask students to find not only the smallest but also the largest value of the function in the definition area, which is already reached at its inner point.

Task 12 (term of the task see above). *Solution.* 1) Whereas $-4 \in (-\infty;1)$, then $f(-4) = (-4)^2 = 16$. 2) The graph of this function consists of fragments of graphs of functions $y = x^2$ and $y = \frac{1}{x}$, that have the common point $(1;1)$ (see figure). 3) Because $E(f)$ is the set of all possible values of variable y, then by the figure and the properties of functions $y = x^2$ and $y = \frac{1}{x}$ we can define that $E(f) = [0;+\infty)$. 4) The line $y = m$ is parallel to the axis Ox. By providing parameter values m and using the figure, we define that one intersection point with the function graph $f(x)$ this line has for all $m \in \{0\} \cup (1;+\infty)$.



Comment. In this task, the most important is the ability of students to justify their reasoning. Paragraph 1) requires the student to explain why he or she chose to calculate the first line in the function definition and not the second. For item 2), the students should be asked to show the graphs of both functions with a dotted line $y = x^2$ and $y = \frac{1}{x}$, and then adjust their joints at appropriate intervals. For paragraph 3), the student needs to explain that the set of values is the set of all possible values of the dependent variable. For paragraph 4), it is necessary for the student to be able to explain that the wanted set of parameter values is determined by the parallel transfer (motion) of a line parallel to the axis Ox. If the student did this task correctly, then teacher can put some additional questions that he or she can answer orally, e.g.: find $f(10)$; find all values m for which the line $y = m$ has with graph of function $y = f(x)$ only two common points or has no common points etc.

Conclusions. We believe that a well-organized thematic preparation for independent assessment will allow teachers to keep their heartbeat on the problems encountered by students in the systematization and repetition of the school mathematics course. We hope that the suggested methodological advice will be of use to all specialists involved in this process. In future publications, we plan to consider the repetition features for all of the above thematic blocks, as well as to regard for each such block a summary test with solutions to the basic tasks and provide methodical comments for them.

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Школьний О. В. Захарійченко Ю. О. Сучасна тематична підготовка до ЗНО з математики в Україні: числа і вирази, функції.

Актуальність досліджень, присвячених тематичній підготовці до ЗНО з математики, в сучасних українських реаліях сумнівів не викликає. Спираючись на багаторічний досвід систематизації та повторення шкільного курсу математики, нами запропоновано розбиття всього курсу математики на 10 тематичних змістових блоків: «Числа і вирази», «Функції», «Рівняння та системи рівнянь», «Нерівності та системи нерівностей», «Текстові задачі», «Елементи математичного аналізу», «Планіметрія», «Стереометрія», «Координати і вектори», «Елементи комбінаторики і стохастики».

У роботі ми наводимо тематичні тести до перших двох змістових блоків («Числа і вирази» та «Функції»), а також відповіді до них. Крім того, ми розв'язуємо опорні задачі цих тестів та подаємо методичні коментарі до цих розв'язань. Ми переконані, що належним чином організована тематична систематизація і повторення шкільного курсу математики дозволить вчителям досягти успіху в підготовці учнів до незалежного тестування з математики.

Ключові слова: *ЗНО з математики, ДПА з математики, тематична підготовка, навчальні досягнення учнів, тематичні тести, опорні задачі, числа і вирази, функції.*

Школьный А. В., Захарийченко Ю. А. Современная тематическая подготовка к ВНО по математике в Украине: числа и выражения, функции.

Актуальность исследований, посвященных тематической подготовке к ВНО по математике, в современных украинских реалиях сомнений не вызывает. Опираясь на опыт систематизации и повторения школьного курса математики, нами предложено разбиение всего курса математики на 10 тематических смысловых блоков: «Числа и выражения», «Функции», «Уравнения и системы уравнений», «Неравенства и системы неравенств», «Текстовые задачи», «Элементы математического анализа», «Планиметрия», «Стереометрия», «Координаты и векторы», «Элементы комбинаторики и стохастики».

В работе мы приводим тематические тесты к первым двум содержательных блокам («Числа и выражения» и «Функции»), а также ответы к ним. Кроме того, мы решаем опорные задачи этих тестов и даем методические комментарии к этим решениям. Мы убеждены, что должным образом организованная тематическая систематизация и повторение школьного курса математики позволит учителям преуспеть в подготовке учеников к независимому тестированию по математике.

Ключевые слова: *ВНО по математике, ГИА по математике, тематическая подготовка, учебные достижения учащихся, тематические тесты, опорные задачи, числа и выражения, функции.*