Lystopadova V., Khalaim D. Application of Lanchester's mathematical laws in military strategy. *Освіта. Інноватика. Практика,* 2023. Том 11, № 8. С. 44-50. https://doi.org/10.31110/2616-650X-vol11i8-007

Lystopadova V., Khalaim D. Application of Lanchester's mathematical laws in military strategy. *Osvita. Innovatyka. Praktyka – Education. Innovation. Practice,* 2023. Vol. 11, No 8. S. 44-50. https://doi.org/10.31110/2616-650X-vol11i8-007

УДК 355.01
DOI: 10.31110/2616-650X-vol11i8-007

Валентина ЛИСТОПАДОВА
Національний технічний університет України
«Київський політехнічний інститут імені Ігоря Сікорського», Україна
http://orcid.org/0000-0002-2549-8381
listopadovavv@gmail.com
Діана ХАЛАЇМ
Національний технічний університет України
«Київський політехнічний інститут імені Ігоря Сікорського», Україна
khalaimdiana@gmail.com

ЗАСТОСУВАННЯ МАТЕМАТЧИНИХ ЗАКОНІВ ЛАНЧЕСТЕРА У ВОЄННІЙ СТРАТЕГІЇ

Анотація. *Математика, як наука про числа, структури та моделі, відіграє важливу роль у багатьох аспектах воєнних операцій та стратегій. Від розрахунку ймовірностей успіху або невдачі військових дій до визначення оптимальних шляхів розгортання військ та ресурсів, математика надає військовим стратегам потужні інструменти для прийняття обґрунтованих та ефективних рішень.*

Одним з важливих аспектів використання математики у військових цілях є розробка стратегій для бойових дій. Від визначення розташування військ та розміру сил до умов проведення операцій, математичні методи та моделі допомагають оптимізувати рішення.

Для визначення ймовірності успіху військових операцій математика використовує теорію ймовірностей та статистику. Це дозволяє оцінити ймовірність досягнення мети, враховуючи різні фактори, такі як військова техніка, розташування супротивника та інші зовнішні умови. Аналіз попередніх військових конфліктів та даних дозволяє статистично оцінити ймовірнісні розподіли та прогнозувати військові події.

Математика також грає важливу роль у розв'язанні задач логістики та розподілу ресурсів. Визначення оптимального маршруту переміщення військ та ресурсів може бути сформульовано як задача оптимізації шляху.

У сфері розвідки та розробки нових технологій також використовуються математичні методи. Криптографія, яка захищає важливу інформацію від несанкціонованого доступу, базується на складних математичних алгоритмах. Математичні моделі також можуть бути використані для симуляцій військових операцій, дослідження впливу нових збройних систем або аналізу траєкторій польоту ракет.

Варто додати, що математичні закони допомагають аналізувати та передбачати результати військових конфліктів, зокрема визначення впливу розміру та ефективності сил противників на ймовірність успіху.

У поданій статті основний акцент зроблено на законах математичних моделей Ланчестера.

Наведено викладки лінійного закону Ланчестера та приклад його застосування.

Розглянуто математичні принципи роботи квадратичного закону Ланчестера на прикладі.

Вказано й висвітлено як ці математичні закони можуть бути застосовані у контексті російського-українсько конфлікту.

Ключові слова: *війна; закони Ланчестера; військові операції; математичні моделі; математичний аналіз.*

Valentyna LYSTOPADOVA
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Ukraine
listopadovavv@gmail.com
http://orcid.org/0000-0002-2549-8381
Diana KHALAIM
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Ukraine
khalaimdiana@gmail.com

APPLICATION OF LANCHESTER'S MATHEMATICAL LAWS IN MILITARY STRATEGY

Abstract. *Mathematics, as the science of numbers, structures, and models, plays an important role in many aspects of military operations and strategies. From calculating the probability of success or failure of military operations to determining the best ways to deploy troops and resources, mathematics provides military strategists with powerful tools to make informed and effective decisions.*

One of the most important aspects of using mathematics in the military is developing strategies for combat operations. From determining the location of troops and the size of forces to the conditions of operations, mathematical methods and models help to optimize decisions.

To determine the probability of success of military operations, mathematics uses probability theory and statistics. This allows us to estimate the probability of achieving the goal, taking into account various factors such as military equipment, enemy location, and other external conditions. The analysis of previous military conflicts and data allows us to statistically estimate probability distributions and predict military events.

Mathematics also plays an important role in solving logistics and resource allocation problems. Determining the optimal route for the movement of troops and resources can be formulated as a path optimization problem.

Mathematical methods are also used in the field of intelligence and the development of new technologies. Cryptography, which protects important information from unauthorized access, is based on complex mathematical algorithms. Mathematical models can also be used to simulate military operations, study the impact of new weapons systems, or analyze missile trajectories.

It is worth adding that mathematical laws help to analyze and predict the outcomes of military conflicts, in particular, to determine the impact of the size and effectiveness of the enemy forces on the probability of success.

In this article, the main focus is on the laws of Lanchester's mathematical models.

The article presents the derivation of Lanchester's linear law and an example of its application.

The mathematical principles of the quadratic Lanchester's law are considered on an example.

It is indicated and highlighted how these mathematical laws can be applied in the context of the Russian-Ukrainian conflict.

Key words: *war; Lanchester's laws; military operations; mathematical models; mathematical analysis.*

Formulation the problem. Given the unstable geopolitical situation in the world and the increased threat of armed conflicts, it is important to have effective methods and tools to assess the likely consequences and outcomes of such conflicts.

The application of Lanchester's Laws, which are based on mathematical models and equations, provides a systematic analysis of the interaction between the parties to a conflict, taking into account the number, effectiveness, and other factors. This makes it possible to predict the probability of success of each party, identify critical factors and strategic decisions that may affect the outcome of the conflict.

Given the complexity and unpredictability of armed conflicts, the application of Lanchester's mathematical laws provides scientifically sound approaches to analysis and strategic decision-making. This can be especially useful for determining the efficiency of resource use, predicting possible conflict scenarios, and calculating optimal strategies.

The application of Lanchester's Laws for military purposes can contribute to a better understanding of the dynamics and consequences of armed conflicts. This can help military strategists, policymakers, and analysts make more informed decisions and ensure more efficient use of resources in military operations.

Analysis of relevant research. Lanchester's Law and ideas related to its application in military conflicts have been covered by a number of scholars. Although warfare technology and tactics have changed significantly since World War I, Lanchester's laws can still be useful for analyzing and understanding some aspects of warfare. Lanchester's Laws highlight general principles such as mass concentration of forces, interaction between forces, and loss of fire. These principles remain relevant regardless of the specific weapons and technologies used. Lanchester's Laws can help analyze and predict various battlefield scenarios and their possible outcomes. This can be useful in planning military operations and making strategic decisions.

The main formulations and ideas on the application of Lanchester's Laws in military conflicts have been highlighted by scholars such as Frederick Lanchester, Robert Hill, Paul Davies, Richard Order and John Fadler. Frederick Lanchester was an English engineer and mathematician who first developed Lanchester's Law. He published his work "Aircraft in Warfare: The Dawn of the Fourth Arm" in 1916 [1]. In this book, he first put forward the theory that the forces of the warring parties in a conflict are proportional to the squares of their numbers. Robert Hill, in turn, studied and expanded Lanchester's concepts in his work "The Mathematical Theory of War" [2]. In this book, he discusses the mathematical aspects of war and the use of mathematical models to analyze military strategies and tactics. Paul Davies continued to study the application of Lanchester's laws in modern military conflicts. His work "Lanchester Equations - Past, Present and Future" examines in detail the use of Lanchester models in military analysis and strategy [3]. The book "Lanchester Equations - Past, Present and Future" uses Lanchester's laws as a theoretical tool for analyzing and modeling military conflicts. Lanchester's laws consider the interaction between enemy forces in a combat situation and determine how various factors, such as the number of forces and weapons, affect the military outcome.

This book explores how Lanchester's equations can be applied to modern military conflicts, taking into account contemporary technology, tactics, and means of warfare. It can be useful for analysts and strategists who are trying to understand and predict the outcomes of military operations in light of modern realities.

The application of Lanchester's Laws to the analysis of current wars can help in studying and understanding the dynamics of conflicts, identifying strategic advantages and disadvantages, and developing rational strategies and tactics to achieve military goals.

Thus, the works of scientists consider the use of mathematical models and Lanchester's laws to analyze the effectiveness of military operations, determine the probability of success and make strategic decisions.

Researchers are actively applying Lanchester's laws and developing new modifications that help to take into account various factors and complexities of modern military conflicts. Their work provides a basis for mathematical analysis and forecasting the outcomes of armed conflicts.

In writing this article, various publications and materials were analyzed, including books, scientific articles and studies related to the application of Lanchester's Laws in military strategy. The author's own thoughts, observations and research on this topic were also taken into account.

The aim of the article. The purpose of the article is to present the basic concepts and principles of Lanchester's Laws, which describe the relationship between forces and the effectiveness of military operations.

Analyze various military conflicts and use Lanchester's Laws to analyze the probability of success, strategic planning and evaluation of results. To highlight the importance of using mathematical models and Lanchester's Laws to improve strategic planning, efficient use of resources, and sound military decision-making.

The purpose of the article is to promote understanding and awareness of the importance of mathematical analysis in the military context and to expand knowledge about the application of Lanchester's laws to analyze possible outcomes of armed conflicts.

Research methods. Analysis of research by scientists and methodologists, analysis of educational programs. To achieve the purpose of the study, we analyzed scientific publications related to a) Lanchester's mathematical laws b) the construction of the quadratic law; c) mathematical methods, tools and technologies that are the basis for military systems

Presenting main material. At the current stage of warfare, where military conflicts are becoming more complex and large-scale, the use of mathematics is an integral part of strategic planning and decision-making. It provides an opportunity to analyze, predict and optimize various aspects of military operations, ensuring more efficient use of resources and reducing the risk of errors [4].

First of all, mathematical models and methods allow us to analyze the situation and predict the development of events. The use of mathematics in military analysis helps to understand the probability of success of military operations, determine optimal strategies, and assess possible outcomes [7]. This provides the command with a sound basis for making important military decisions.

Mathematics is indispensable in determining the efficient use of resources. By optimizing the distribution of military forces, weapons, equipment, and other resources, we ensure maximum efficiency on the battlefield. Mathematical methods help to determine the optimal number and deployment of military units, as well as to allocate the necessary resources among them [5].

In addition, the use of mathematics in military operations allows us to identify and analyze the enemy's vulnerabilities, including its logistics, communications, and force deployment. This helps to develop strategies aimed at effectively exploiting these weaknesses and gaining an advantage in the conflict [11].

Let's consider a situation where it is necessary to predict whether it is possible to go on the offensive against the enemy or retreat to fight another day. We have a situation where the enemy is twice as numerous as the opponent's troops, but is much less trained in warfare. The main question is whether mathematics can be used to predict the outcome of such a battle and help decide what to do in such a situation.

Mathematical models are useful tools for analyzing real-world situations. They help simplify complex processes and allow us to calculate potential outcomes.

The first step in applying a good mathematical model to any system is to understand the basic processes that make the system work [12]. In this case, the overall system is a battle, and the main interaction is the fight between individual soldiers [10]. For simplicity, let's assume that each soldier on the front line fights the opposite soldier on the other side only when one wins, the loser is instantly replaced by another soldier from that army. This creates a static front line between the armies. Thus, soldiers located further from the front do not actually fight any battles until they reach this line. This small detail is very important.

Let's add two new parameters α and β. They are needed to determine how well trained and armed each army is. α represents how effective each soldier is at defeating the enemy, where a value of 1 means that each soldier in the line defeats one enemy per unit of time, and none at 0. The β represents the same, but for the enemy. These parameters depend on various factors. Since troops get tired during a battle, the parameters will depend to some extent on time, and if we take into account the morale of the soldiers, it will probably also depend on the size of your own army and the enemy army. For the sake of simplicity, let's assume that they are constant: α = const, β = const. This will be one of the main assumptions to simplify the model.

We can now use these constants to construct a differential equation that describes how the size of each army changes over time. Let's use the notation A(t) to denote the size of one army and the notation B(t) to denote the size of the enemy at a certain time. The rate of change of the army size is denoted as $\frac{dA}{dt}$. The number of soldiers lost per unit of time should be equal to the number of enemy soldiers attacking that army multiplied by the effectiveness of those soldiers, which is the number of soldiers defeated per unit of time. So:

$$\frac{dA}{dt} = -\beta N, \tag{1}$$

where N is the number of enemy soldiers on the front line. We put a minus sign because we lose troops rather than gain them.

For the enemy, we get a similar equation, but with the coefficient α:

$$\frac{dB}{dt} = -\alpha N, \tag{2}$$

Since both armies have the same number of soldiers on the line at the same time, N is the same in these two equations. It is important to note that both armies are shrinking at a constant rate. This is called Lanchester's linear law because if we plot each army against time, the resulting trend is a straight line. The gradient of this line is proportional to the effectiveness of the enemy soldiers, and the point where each line crosses the ordinate axis is the initial size of each army (Fig. 1):

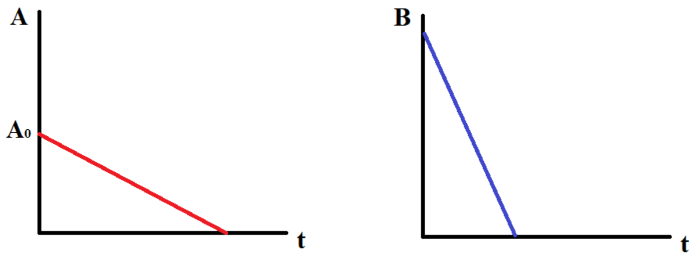

Fig 1. Image of Lanchester's linear law

Suppose that both armies are ready to fight to the end. The losing side is the army that will be destroyed first, so their line will be the first to reach zero. We know that the enemy army is twice the size of the opponent's army: $B_0 = 2A_0$, so to win, the soldiers must be more than twice as effective as the enemy. We can achieve this result algebraically.

Let's divide the two previous equations by one:

$$\frac{(\frac{dA}{dt})}{(\frac{dB}{dt})} = \frac{\beta}{\alpha}$$

By integrating this combined equation, we get the following:

$$\int_{A_0}^{A} \alpha dA = \int_{B_0}^{B} \beta dB \rightarrow \alpha(\mathrm{A} - \mathrm{A}_0) = \beta(\mathrm{B} - B_0),$$

where A_0 and B_0 are the initial size of each army.

Since we know what we need to win the battle, we are looking for conditions under which A > 0 and B = 0. Under these conditions, the enemy will be destroyed. Consider this case specifically for this equation $\alpha(\mathrm{A} - \mathrm{A}_0) = \beta(\mathrm{B} - B_0)$. The enemy will be destroyed if:

$$\alpha\mathrm{A}_0 > \beta B_0.$$

The result of multiplying the efficiency by $\alpha\mathrm{A}_0$ is the combat power of the army $\mathrm{F_A}$, and the army that has the highest result of this coefficient will win the battle. Let's compare the strength of the armies:

$\mathrm{F_A} > \mathrm{F_B}$ → wins the army A

$\mathrm{F_B} > \mathrm{F_A}$ → opponent B wins

Since the enemy has twice as many units ($B_0 = 2\mathrm{A}_0$), his opponent must be twice as efficient ($\alpha = 2\beta$) in order to have equal combat power $\alpha\mathrm{A}_0 = \beta B_0$. Thus, the enemy will be defeated if one soldier is equal in effectiveness to at least three enemy soldiers ($\alpha = 3\beta$). That is, if for every soldier of Army A there are two enemies of Army B, but each soldier of Army A is worth three of Army B, then it is clear that the troops of A will win.

Lanchester's linear law describes such simple scenarios, where the number of soldiers fighting is fixed at one time, since the limited front line does not allow Army B to surround Army A and defeat it. If we talk about the encirclement of one army by another, we have a less obvious result.

Let's consider the following situation, in which we pay attention to Lanchester's quadratic law. Suppose we have an army A of 1000 men crossing a large flat surface, and an army B of 2000 men is heading towards them. So, again, Army B outnumbers Army A by two to one, but they have better weapons that allow them to respond to an attack at three times the enemy's speed, which again makes them three times more effective. Given the same conditions, Army A will be defeated. Let's look at why.

The key difference lies in the terrain of the battlefield. On a straight battlefield, fighting takes place only between those on the front lines of the battle, with the rest having to wait. However, on a flat surface, there are no restrictions that would limit the battle and the ability of soldiers to attack from a distance means that at any given time, every soldier can attack the enemy and also be attacked. That is, no one waits for their turn as in the previous case, the entire army of A competes with the entire strength of the army of B at once.

Let's form some equations similar to the previous ones that describe how the size of each army changes over time. Just as in the straight line, the rate at which each army shrinks will be equal to the number of enemy soldiers attacking at the same time multiplied by the efficiency of those attackers. The only difference is that the number of attackers is no longer constant and the front line is not limited:

$$\frac{dA}{dt} = -\beta B,$$

$$\frac{dB}{dt} = -\alpha A.$$

Consequently, the size of each army does not follow a linear trend over time. This phenomenon cannot be represented on a graph as a straight line because the gradient is no longer constant.

If we combine these equations again and integrate the result, we get the following equation:

$$\frac{(\frac{dA}{dt})}{(\frac{dB}{dt})} \rightarrow \frac{dA}{dB} = \frac{\beta B}{\alpha A},$$

$$\int_{A_0}^{A} \alpha A dA = \int_{B_0}^{B} \beta B dB \rightarrow \alpha(A^2 - A_0{}^2) = \beta(B^2 - B_0{}^2).$$

We can see the square of the number of soldiers in each army. This is Lanchester's law of squares.

Let's consider what conditions must be met for Army A to win. For this to happen, we need: A > 0 and B = 0. When we plug these constraints into the equation, we get the following constraint:

$$\alpha A_0{}^2 > \beta B_0{}^2.$$

$\alpha A_0{}^2 = F_A$ – the new fighting power of the army. Since the model assumes that whichever side has the highest value wins the battle, this is almost identical to the fighting strength determined in the linear case, except that now it is the square of the army's size. This means that having more soldiers gives you a much better chance of winning. For example, if you double the size of an army, their fighting power would be four times greater. This is the reason why Army A was defeated, because the numerical advantage of Army B squared was greater than the efficiency advantage that Army A had:

$$\alpha A_0{}^2 \qquad \beta B_0{}^2$$
$$(\alpha = 3\beta)\ (B_0 = 2A_0)$$
$$3\beta A_0{}^2 \qquad 4\beta A_0{}^2$$
$$F_A \quad < \quad F_B$$

To demonstrate the power of the quadratic law, consider what would happen if the two sides were identical in effectiveness, but one had a slight advantage in numbers. Suppose Army C has 1000 men and Army D has 700. We can assume that Army C will win, but lose most of their troops in the process, but thanks to the law of the square, Army C actually wins the battle with 714 soldiers, which is still a very large part of their original army:

$$\alpha(A^2 - A_0{}^2) = \alpha(B^2 - B_0{}^2)$$
$$(\text{We are looking for A since B} = 0)$$
$$A_0 = 1000 \qquad B_0 = 700$$
$$A^2 = 1000^2 - 700^2$$
$$A = \sqrt{1000^2 - 700^2} = 714$$

Such a small number of advantages have such a huge impact on who wins the battle. At a fundamental level, this is because the strength of an army's attack is directly proportional to its size:

$$-\frac{dB}{dt} = \alpha A.$$

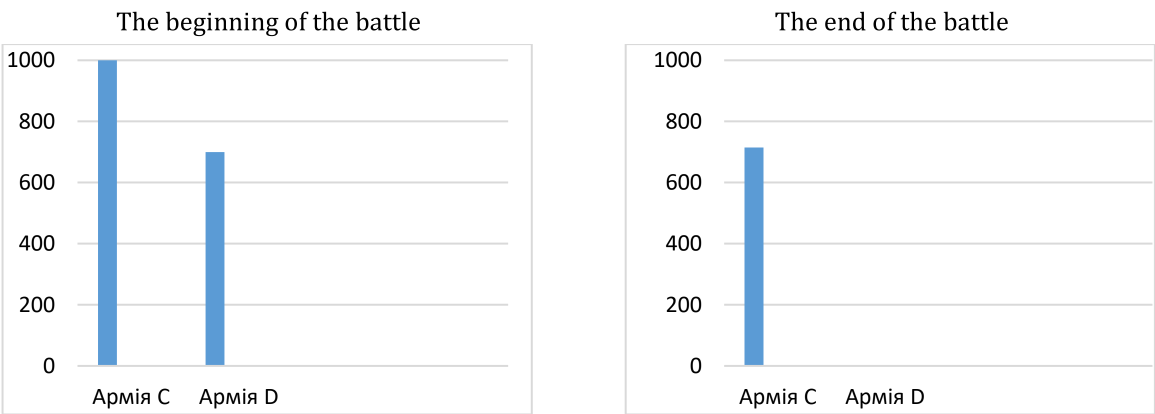

Fig. 2. Diagram of the quadratic law for the same efficiency and different number of troops

At the beginning of a battle, a larger force does a lot of damage to a smaller force, while a smaller force does less damage because, as noted earlier, the strength of an attack is proportional to the number of people [8]. This means that a smaller force will lose more people than a larger force, increasing the difference in numbers. This process is repeated over and over again with the difference in size, and as a result, the difference in strength becomes larger and larger until the smaller force is completely destroyed.

As the smaller force reduces its ability to harm the larger force, the larger force also reduces its ability to harm the smaller force, making the power imbalance even more pronounced [9]. This effect eventually combines with a small size difference in the beginning, leading to a much larger size difference later.

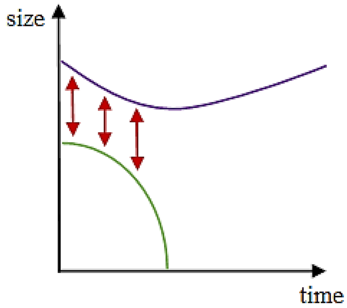

Fig. 3. Graph of the quadratic law for the same efficiency and different number of troops

All of the above is the result of the fact that every soldier could immediately engage the enemy.

The English engineer Friedrich Lanchester came up with these equations in 1916 during World War I, where long-range weapons were used.

These general mathematical models can give a rough idea of what is likely to happen. The fewer assumptions there are, the more reliable the result. They also help to provide a much better understanding of the general principles that apply, which is ultimately an incredibly valuable tool.

The above calculations can be applied to modern military conflicts, including the current Russian-Ukrainian conflict, which is relevant for our country. As already mentioned, Lanchester's first law, also known as the law of firepower, describes the relationship between the firepower of two enemy sides and the change in the number of their troops over time. The following mathematical model can be considered for the Russian-Ukrainian conflict:

Let N1(t) be the number of military units of the Ukrainian army at time t, and N2(t) be the number of military units of the Russian army at time t. The firepower of the Ukrainian army will be denoted as F1(t), and the firepower of the Russian army will be denoted as F2(t).

The law of firepower can be expressed by the following formula:

$$\frac{\mathrm{dN1(t)}}{dt} = -\mathrm{k2} * \mathrm{F2(t)}$$

$$\frac{\mathrm{dN2(t)}}{dt} = -\mathrm{k1} * \mathrm{F1(t)}$$

where dN1(t)/dt and dN2(t)/dt are the rate of change in the number of military units of the Ukrainian and Russian armies relative to time t, k1 and k2 are the proportionality coefficients that reflect the impact of the enemy's firepower on the reduction in the number of military units, F1(t) and F2(t) are the firepower of the Ukrainian and Russian armies relative to time t.

These equations indicate that the change in the number of military units on each side is proportional to the enemy's firepower. The greater the enemy's firepower, the faster the number of enemy military units decreases [6].

This mathematical model can be used to analyze and predict the dynamics of the number of military forces of the Ukrainian and Russian armies during the conflict.

Analyzing the above mathematical derivations of Lanchester's first law for the Russian-Ukrainian conflict and without taking into account extraneous factors, the following observations can be made:

1. Depending on the firepower of each side, the number of military forces will decrease over time. The greater the firepower of one side, the faster the number of enemy forces will decrease.

2. The patterns of dependence between firepower and changes in the number of military forces allow us to predict the general trend. For example, if the firepower of the Russian army exceeds the firepower of the Ukrainian army, we can expect the number of Ukrainian troops to decline faster than the number of Russian troops.

3. The temporal distribution of changes in the number of military forces may be uneven depending on the firepower of the parties. For example, during intense hostilities, the reduction in numbers may be greater than during calm periods.

It should be borne in mind that these conclusions are based only on mathematical models and assumptions, and do not take into account real strategic, tactical, political and economic factors that can significantly affect the dynamics of the number of military forces of the Ukrainian and Russian armies during the conflict.

Conclusions and prospects for further research. Mathematics helps to determine strategic decision-making, calculate the optimal deployment of military resources and the distribution of forces on different fronts. This helps to increase the efficiency of military operations and reduce losses.

The use of mathematics also helps to develop strategies to counter enemy actions, analyze and predict the possible consequences of various conflict scenarios, and assess the effectiveness of various types of weapons and technologies.

The use of mathematics in the military is essential for making informed decisions, minimizing risks, and achieving strategic goals. It allows us to understand complex relationships, identify weaknesses, and develop optimal approaches to warfare.

The use of mathematical models, in particular Lanchester's laws, plays an important role in military operations. They allow analyzing and predicting the dynamics of the number of military forces, determining the impact of firepower and other factors on the outcome of hostilities.

References

1. Lanchester, F. W. Aviatsiia na viini: Svitanok Chetvertoho Reikhu. Konstebl, 1916. S. 2-4.
2. Hill R. L. (1950). Matematychna teoriia viiny. S. 10-12.
3. Davis P. (2018). Rivniannia Lanchestera – mynule, teperishnie, maibutnie. S. 20.
4. Wein, L. M. Matematyka na viini. Povidomlennia Amerykanskoho matematychnoho tovarystva, 2008. S. 210-218.
5. Kress, M. Matematyka konfliktu. Vydavnytstvo Prinstonskoho universytetu, 2010. S. 10
6. Tolk, A., Diallo, S. Y., Ryzhov, I. O., Yilmaz, L., & Ferris, M. (Eds.). Materialy Zymovoi konferentsii z modeliuvannia 2017. IEEE Press, 2017.
7. Hill, R. J., & Davis, P. W. Vstup do doslidzhennia operatsii u viiskovomu zastosuvanni. John Wiley & Sons, 1971.
8. Kress, M., & Tolk, A. (Eds.). Matematychne modeliuvannia viiny ta boiovykh dii. Sprinher, 2019.
9. Order, R. V. Prykladna matematyka: Tilo i dusha: Tom 1: Pokhidni ta heometriia v IRn. Springer, 1991.
10. Feidler, J. Vstup do matematyky doslidzhennia operatsii. CRC Press, 1995.
11. Blythe, P. T., & Giordano, F. Matematychni modeli ta metody doslidzhennia viiskovykh operatsii. CRC Press, 2021.
12. Szechtman, R., & Voss, S. (Eds.). Posibnyk z doslidzhennia viiskovykh ta oboronnykh operatsii. CRC Press, 2022.