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#### MODERN THEMATIC PREPARATION FOR EIA IN MATHEMATICS IN UKRAINE: COORDINATES AND VECTORS, ELEMENTS OF COMBINATORICS AND STOCHASTICS

##### ABSTRACT

**Formulation of the problem.** In modern conditions, the relevance of research on thematic preparation for the IEA in mathematics is undeniable. External Independent Assessment is now the main instrument of evaluation of the quality of mathematical training for Ukrainian senior school students. In particular, it is used for conducting the State Final Attestation of academic achievements of graduates, as well as as a tool for competitive selection of applicants to Ukrainian high education institutions. Thus, we have no doubt about the importance and the need for research on various aspects of preparation for the EIA in mathematics. One such aspect is the thematic repetition of the school mathematics course.

**Materials and methods.** To achieve this goal we apply some empirical methods: observation of the training process of the students during their studying on training courses for the EIA in mathematics and analysis of the results of their achievements. The research also used a set of methods of scientific cognition: a comparative analysis to find out different views on the problem and determine the direction of research; systematization and generalization for the formulation of conclusions and recommendations; generalization of author's pedagogical experience and observations.

**Results.** Based on the author's experience of systematization and repetition of the school mathematics course in preparation for IEA, we propose to divide the entire mathematics course into 10 logical content blocks. In this article, we provide thematic tests of the content blocks «Coordinates and vectors», «Elements of combinatorics and stochastics», as well as answers to them. We also solve some of the basic tasks of these tests and give some methodical comments on these solutions. The vector and coordinate methods very often make much easier the process of geometric problems solving in comparison with traditional methods. Statistical and probabilistic methods are used as a means of modeling the processes and phenomena of the real world, and therefore, their study contributes to the formation of the outlook of the child.

**Conclusions.** We believe that well-organized thematic training for EIA and SFA in mathematics will allow teachers to overcome the problems encountered by students in the systematization and repetition of the school mathematics course. This publication completes a series of our articles on modern thematic preparation for the EIA in mathematics. In them, we outlined our vision for the methodology of its organization, as well as shared our didactic materials and methodological tips.

**KEY WORDS:** IEA in math, SFA in math, thematic preparation, educational achievements of students, thematic tests, basic tasks, coordinates, vectors, combinatorics, stochastics.

##### INTRODUCTION

**Formulation of the problem.** External Independent Assessment (EIA) is now the main instrument of evaluation of the quality of mathematical training for Ukrainian senior school students. In particular, it is used for conducting the State Final Attestation (SFA) of academic achievements of graduates, as well as as a tool for competitive selection of applicants to Ukrainian high education institutions. Thus, we have no doubt about the importance and the need for research on various aspects of preparation for the EIA in mathematics. One such aspect is the thematic repetition of the school mathematics course.

Based on our experience in training students to EIA, during this repetition we divide the whole mathematics course into 10 thematic blocks: «Numbers and expressions», «Functions», «Equations and systems of equations», «Inequalities and systems of inequalities», «Text problems», «Elements of mathematical analysis», «Geometry on the plane», «Geometry in the space»,

«Coordinates and vectors», «Elements of combinatorics and stochastics». This division that allows repeated repetition of the same material throughout the preparation process for the EIA. For example, transformation of rational expressions and action over numbers are repeated during the study of all thematic blocks 1-10. This permits the teacher constantly to keep the student in a tone, when he or she would forget something, but can't do this, because proposed thematic training system doesn't allow it.

**Analysis of current research.** The problem of preparing students for EIA in mathematics is systematically reviewed in different scientific and pedagogical publications. Always print the results of their investigations in this area of research Valentyna Bezv, Mykhailo Burda, Hryhoriy Bilyanin, Olga Bilyanina, Olga Vashulenko, Larysa Dvoretzka, Oxana Yergina, Oleksandr Ister, Vadym Karpik, Arkadiy Merzlyak, Yevgen Nelin, Victor Repeta, Oleksiy Tomaschuk, Mykhailo Yakir and others. During more than last 15 years, our author's team has been continuously working to provide methodological support for the process of preparation for the EIA in mathematics. The theory and methodology of evaluating the academic achievement of senior school students in Ukraine is given in the monograph (Shkolnyi 2015). For the training and systematization of the school mathematics course, we use the methodological set of books (Zakhariichenko et al 2019a) and (Zakhariichenko et al 2019b). Previously, we have considered certain aspects of thematic preparation for independent testing, but since then the contingent of EIA participants has changed significantly, as well as the methodological views of our author's team on this problem are also developed. This article is the final article of the series of our articles devoted to this problem that was started in the article (Shkolnyi & Zakhariichenko 2019).

**The purpose of the article.** The purpose of this article is to give methodological advice to teachers and tutors that concerned in qualitative thematic preparation to EIA in mathematics. In particular, we present here two thematic tests related to the topics «Coordinates and vectors», «Elements of combinatorics and stochastics», and also provide a solution of the some basic tasks of these tests with methodical comments to them.

## RESEARCH METHODS

To achieve our goal we use in this paper some theoretical methods, such as an analysis of methodological literature on the research subject. Also we apply some empirical methods: observation of the training process of the students during their studying on training courses for the EIA in mathematics and analysis of the results of their achievements. The research also used a set of methods of scientific cognition: a comparative analysis to find out different views on the problem and determine the direction of research; systematization and generalization for the formulation of conclusions and recommendations; generalization of author's pedagogical experience and observations.

## RESULTS AND DISCUSSIONS

The vector and coordinate method is a powerful tool for solving a wide class of mathematical problems. It is known that many geometry problems on the plane and in the space are solved much easier using coordinates and vectors than traditional methods. Therefore, it is quite natural for students to become familiar with this topic in the school course of mathematics. It is also natural to systematize and repeat tasks on using of coordinates and vectors in preparation for the EIA in mathematics. In doing so, it is important to focus on the numerous applications of this topic to solve those problems that are more difficult to solve by traditional methods.

Combinatorics, probability theory and mathematical statistics (stochastics) are an important component of forming an adequate outlook for a young person. Therefore, studying their elements in the school course of mathematics is also natural. Unfortunately, often the study of this material is rather formal – the students solve many abstract problems in the application of formulas. We emphasize the practical importance of this topic and encourage teachers to use as many real-life tasks as possible. It is clear that real life is very complicated, and the probabilistic models of real processes and phenomena considered at school are mainly their simplified models. However, even such simplified models allow students to understand the importance of probabilistic and statistical methods and teach them to apply these methods in practice.

For these reasons, the task on using coordinates, vectors, and elements of combinatorics and stochastics have to be present in the mathematics test. At the same time, both the tasks for the practical application of these sections and the theoretical tasks are important. We believe that open-ended task with full explanation give the best possibility to achieve this goal. We can be limited by tasks of this form in student's preparation to the EIA, because they are the most effective for teaching mathematics and feedback. However, after finishing each of the 10 thematic blocks, it is natural to perform a diagnostic thematic test in which to use all forms of test tasks inherent in the EIA math test. Let's look at two such tests below. The first refers to using coordinates and vectors in geometry and the second to solving problems on stochastics and combinatorics.

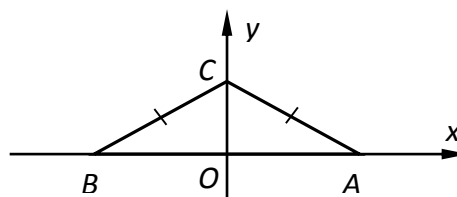
*Thematic test «Coordinates and vectors».*

**Tasks 1-7 have five answer choices, only one of which is correct. Choose the correct answer, in your opinion.**

1. Specify the point that belongs to the second coordinate quarter.

A	B	C	D	E
$A(1; 2)$	$B(-1; 2)$	$C(2; 1)$	$D(-2; -1)$	$E(2; -1)$

2. Determine the coordinates of the vertex A of the isosceles triangle ABC, shown in the figure, if  $B(-8; 0)$ .



A	B	C	D	E
(0; 4)	(0; -8)	(8; 0)	(-8; 0)	(0; 8)

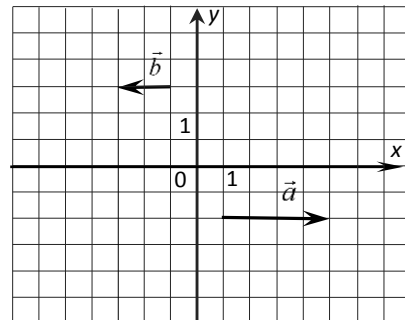
3. Determine the coordinates of the center of the circle given by the equation  $(x-3)^2 + y^2 = 4$ .

A	B	C	D	E
(0; -3)	(0; 3)	(-3; 0)	(3; 0)	(3; 4)

4. Determine the coordinates of the vector  $\vec{BA}$ , if  $A(-1; 5)$ ,  $B(3; -3)$ .

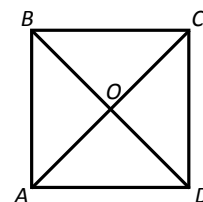
A	B	C	D	E
(1; 1)	(2; 2)	(4; -8)	(-2; -2)	(-4; 8)

5. On the figure is drawn collinear vectors  $\vec{a}$  and  $\vec{b}$ . Express vector  $\vec{a}$  through the vector  $\vec{b}$ .



A	B	C	D	E
$\vec{a} = -\frac{1}{2}\vec{b}$	$\vec{a} = -2\vec{b}$	$\vec{a} = -\vec{b}$	$\vec{a} = 2\vec{b}$	$\vec{a} = \frac{1}{2}\vec{b}$

6. On the figure is drawn a square  $ABCD$ ,  $O$  is the point of intersection of its diagonals. Specify a vector that is equal to the vector  $\vec{OD} + \vec{OC}$ .



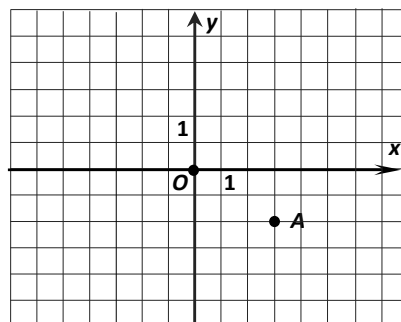
A	B	C	D	E
$\vec{BC}$	$\vec{DA}$	$\vec{BA}$	$\vec{BD}$	$\vec{DC}$

7. In a rectangular coordinate system the point  $M(5; 4; 7)$  is given. Specify a point symmetric to the point  $M$  with respect to the axis  $Oy$ .

A	B	C	D	E
$(-5; -4; -7)$	$(0; 4; 0)$	$(5; -4; 7)$	$(-5; 4; -7)$	$(5; 0; 7)$

In the task 8 for each of the three rows of data marked with numbers, select the one correct, in your opinion, variant marked with a letter.

8. On the figure is drawn a rectangular system of coordinates  $Oxy$ , in which points  $O(0; 0)$  and  $A(3; -2)$  are given. Match the beginning of the sentence (1-3) and its end (A-E) so that the correct statement will be formed.



Beginning of the sentence

- 1 Point  $(-3; 2)$
- 2 Point  $(2; -3)$
- 3 Point  $(3; 2)$

End of the sentence

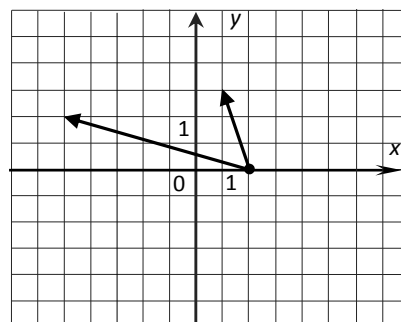
- A is symmetric to the point  $A$  with respect to the axis  $Oy$
- B is symmetric to the point  $A$  with respect to the axis  $Ox$
- C is symmetric to the point  $A$  with respect to the point  $O$
- D belongs to the straight line  $x = 2$
- E belongs to the straight line  $y = 3$

Solve tasks 9-11. Record the numeric answers you received in decimal or integer.

9. The vertexes  $A(-3; 4; 1)$ ,  $B(3; -2; -7)$ ,  $C(5; 2; -9)$  of triangle  $ABC$  are given. Define: 1) the applique of middle of segment  $BC$ ; 2) the square of the median length of this triangle drawn from the top  $A$ .
10. Find the scalar product of the vectors shown in the figure.
11. The circle centered at the point  $O(x_0; y_0)$  is given by the equation  $x^2 + y^2 + 10x - 16y + 7 = 0$ . Find the value of the product  $x_0 \cdot y_0$ .

**Solve the task 12. Write down sequential logical actions and explanations of all stages of task solving, make reference to the mathematical facts from which one or another statement follows. If necessary, illustrate the task solving with drawings, graphs, etc.**

12. A rectangular parallelepiped  $ABCD A_1 B_1 C_1 D_1$  with  $AB = 6$ ,  $AD = 10$ ,  $AA_1 = 8$  is given. Point  $O$  is the point of intersection of the diagonals of the face  $ABCD$ , point  $L$  is the middle of  $B_1 C_1$ . 1) Express vector  $\vec{LO}$  through vectors  $\vec{BA}$ ,  $\vec{BC}$  and  $\vec{BB_1}$ . 2) Find the angle between vectors  $\vec{LO}$  and  $\vec{B_1 D}$ .



Answers to the test «Coordinates and vectors»

1	2	3	4	5	6	7	8	9	10	11
B	C	D	E	B	A	D	1 - C, 2 - D, 3 - B	1) -8; 2) 146	13	-40

12. 1)  $\vec{LO} = \frac{1}{2}\vec{BA} - \vec{BB_1}$ ; 2)  $\arccos \frac{41}{5\sqrt{146}}$

Solutions and comments to tasks of the test «Coordinates and vectors».

**Task 8** (term of the task see above). *Solution.* We analyze alternatives from the right column (end of sentences). **A.** The point we need to find has coordinates  $(-3; -2)$ . This completion of the sentence does not correspond to any beginning. **B.** The point we need to find has coordinates  $(3; 2)$ . This completion of the sentence corresponds to the beginning **3**. **C.** The point we need to find has coordinates  $(-3; 2)$ . This completion of the sentence corresponds to the beginning **1**. **D.** This completion of the sentence corresponds to the beginning **2**. **E.** This completion of the sentence does not correspond to any beginning. So the correct answer is: **1 - C, 2 - D, 3 - B**.

*Comment.* This way of solving this problem is not typical. Traditionally, we analyze alternatives from the left column and find alternatives from the right column that correspond to them. However, in this case, such method is obviously more complicated. In addition, task 8 examines how well students understand the concept of symmetry (with respect a point and with respect to a straight line), which has many practical applications.

**Task 12** (term of the task see above). *Solution.* 1) By the rule of adding vectors  $\vec{LO} = \vec{LB_1} + \vec{B_1 B} + \vec{BO}$ . Since  $\vec{LB_1} = -\frac{1}{2}\vec{BC}$ ,  $\vec{B_1 B} = -\vec{BB_1}$ ,  $\vec{BO} = \frac{1}{2}(\vec{BA} + \vec{BC})$ , then after transformation we obtain  $\vec{LO} = \frac{1}{2}\vec{BA} - \vec{BB_1}$ . 2) Let's introduce the rectangular system of coordinates with beginning point in point  $B$ , which positive directions of axes  $x$ ,  $y$  and  $z$  are defined by vectors  $\vec{BA}$ ,  $\vec{BC}$  and  $\vec{BB_1}$  respectively. Then, according to the problem condition, the points  $L$ ,  $O$ ,  $B_1$ ,  $D$  have the following coordinates:  $L(0; 5; 8)$ ,  $O(3; 5; 0)$ ,  $B_1(0; 0; 8)$ ,  $D(6; 10; 0)$ .

Thus,  $\vec{LO}(3; 0; -8)$ ,  $\vec{B_1 D}(6; 10; -8)$ . Let  $\varphi$  is the angle between  $\vec{LO}$  and  $\vec{B_1 D}$ . Using scalar product, we obtain that  $\cos \varphi = \frac{\vec{LO} \cdot \vec{B_1 D}}{|\vec{LO}| \cdot |\vec{B_1 D}|} = \frac{18 + 0 + 64}{\sqrt{9 + 0 + 64} \cdot \sqrt{36 + 100 + 64}} = \frac{41}{5\sqrt{146}}$ .

*Comment.* This task illustrates the use of vector and coordinate methods to solve geometric problems. The first part of Task 12 is purely technical and tests how well the student has mastered linear operations on vectors. The second part, in fact, shows how to find the angle between the passing lines in space. For an arbitrary parallelepiped to solve such a problem by traditional methods is difficult, if possible at all. Solving such problems broadens the student's mathematical outlook, enriches his (her) erudition, and promotes qualitative preparation for the EIA in mathematics.

Thematic test «Elements of combinatorics and stochastics».

**Tasks 1-7 have five answer choices, only one of which is correct. Choose the correct answer, in your opinion.**

1. Which of the following may be the probability value of some random event?

A	B	C	D	E
$-\frac{1}{2}$	$\frac{5}{3}$	$\sqrt{2}$	$\frac{3}{5}$	2

2. Among all residents of the city of Kiev randomly choose one resident. What is the probability that the height of this person exceeds 5 m?

A	B	C	D	E
1	0,8	0,5	0,2	0

3. In the word BANANA randomly select one letter. What is the probability that this letter will be letter N?

A	B	C	D	E
$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{5}$

4. In the first department there are 6 managers, in the second department there are 5 managers. From the first and the second departments choose one manager for foreign business trip. How many choices are there for these two managers?

A	B	C	D	E
2	11	30	55	110

5. Each of the three students was asked, "How many funny smiles have you been sending in the last hour?" The following answers were received: 9; 2; 4. Determine the median of this sample.

A	B	C	D	E
4	5	2	7	9

6. The smallest element of a given sample is 30 and the sample size is 20. Specify a number that *can be* the arithmetic mean of that sample.

A	B	C	D	E
10	20	30	40	50

7. Nine graduates of driving school will take the driving test. Each graduate have to drive one time with the examiner in the car. To do this, a list of graduates will be required to take the exam. How many different lists of these graduates can you make? Specify the formula, by that all the options can be calculated.

A	B	C	D	E
$C_9^2$	$9!$	$10!$	$2 \cdot 9$	$A_9^2$

In the task 8 for each of the three rows of data marked with numbers, select the one correct, in your opinion, variant marked with a letter.

8. There are three yellow tokens numbered 1, 2 and 3 and two blue tokens numbered 3 and 4 in the urn. One token is randomly removed from the urn. Match the random event (1–3) and its probability (A–E).

Random event

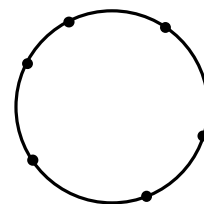
Probability of random event

- 1 Blue token is removed
- 2 Number 4 token is removed
- 3 Token with odd number is removed

- A  $\frac{3}{5}$
- B  $\frac{1}{2}$
- C  $\frac{1}{5}$
- D  $\frac{1}{3}$
- E  $\frac{2}{5}$

Solve tasks 9-11. Record the numeric answers you received in decimal or integer.

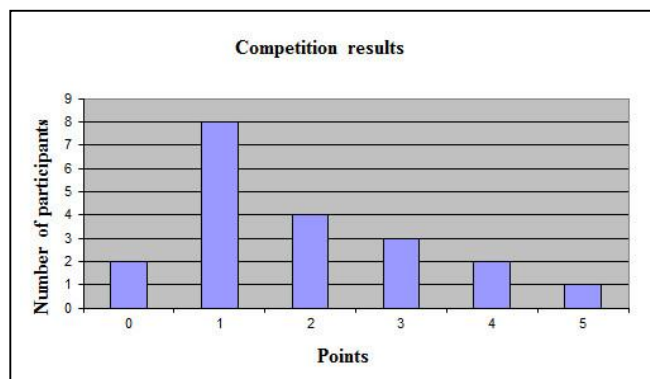
9. Determine: 1) the number of even two-digit positive integers; 2) the probability that the randomly named pair of two-digit positive integer second digit is 8.
10. Six points are marked on the circle (see figure). How many different triangles with vertices at these points can be constructed?
11. Ms. Tamara chooses a costume for a formal meeting. For such events she has 5 different jackets, 6 different skirts, 8 different pants and 10 different blouses. She plans to wear a jacket, a blouse along with a pants or a skirt. How many different costumes for the formal meeting are there for Ms. Tamara?



Solve the task 12. Write down sequential logical actions and explanations of all stages of task solving, make reference to the mathematical facts from which one or another statement follows. If necessary, illustrate the task solving with drawings, graphs, etc.

12. During sports competitions, participants were asked to perform 5 shots of the ball in the basket. There was 1 point for each hit. The results of the competition are shown in the diagram. Determine: 1) the total number of participants; 2) the arithmetic mean of the number of points scored; 3) mode; 4) the median; 5) how many times the participants who scored 2 points were more than the participants who scored 5 points

Answers to the test «Elements of combinatorics and stochastics»



1	2	3	4	5	6	7	8	9	10	11
D	E	C	C	A	D	B	1 – E, 2 – C, 3 – A	1) 45; 2) 0,2	20	700

12. 1) 20; 2) 1,9; 3) 1; 4) 1,5; 5) 4.

Solutions and comments to tasks of the test «Elements of combinatorics and stochastics».

**Task 2** (term of the task see above). *Solution.* Since there are no people 5m tall, the required probability is zero and the correct answer is E.

*Comment.* Such tasks are very important because they test not only the student's technical skills of calculating probabilities, but his understanding the essence of this concept. Indeed, for successful practice, it is important to distinguish which events are plausible and always true, which events are impossible and never occur, and which events are accidental and can be both performed and not performed.

**Task 11** (term of the task see above). *Solution.* Ms. Tamara can wear for the meeting jacket, blouse and pants, or jacket, blouse and skirt. According to the combinatorial rules of addition and multiplication, we have  $5 \cdot 10 \cdot 8 + 5 \cdot 10 \cdot 6 = 700$  ways of forming a costume for Ms. Tamara at this meeting.

*Comment.* In real life, we are often faced with the possibility of choosing one of several options. Quite often, we do not imagine how large the number of such options can be, and therefore we cannot correctly estimate the complexity of this choice. Knowledge of combinatorics can overcome such difficulties and promotes adequate perception of the surrounding reality. Thus, the teacher should pay special attention to similar tasks both during teaching of mathematics at school and during preparation of graduates for the EIA test.

## CONCLUSIONS AND PROSPECTS OF FURTHER RESEARCH

Vector and coordinate methods are important components of mathematical preparation for the modern student. They allow them to look at classical geometric problems from other positions, and in many cases greatly simplify the solution of these problems. Therefore, the teacher should pay due attention to these methods during preparing for EIA in mathematics. Probabilistic and statistical tasks perform an equally important ideological function. They have a pronounced practical direction and contribute to the formation of an adequate picture of the world of modern graduates. Therefore, the presence of combinatorial and stochastic problems in the mathematics test is natural and usually important.

We believe that well-organized thematic training for EIA and SFA in mathematics will allow teachers to overcome the problems encountered by students in the systematization and repetition of the school mathematics course. This publication completes a series of our articles on modern thematic preparation for the EIA in mathematics. In them, we outlined our vision for the methodology of its organization, as well as shared our didactic materials and methodological tips. We hope that the materials provided will be useful for teachers to ensure that the graduates are properly trained to standardized mathematics testing.

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## СУЧАСНА ТЕМАТИЧНА ПІДГОТОВКА ДО ЗНО З МАТЕМАТИКИ В УКРАЇНІ: КООРДИНАТИ І ВЕКТОРИ, ЕЛЕМЕНТИ КОМБІНАТОРИКИ І СТОХАСТИКИ

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### Анотація.

**Формулювання проблеми.** У сучасних умовах актуальність досліджень щодо тематичної підготовки до ЗНО з математики незаперечна. Зовнішнє незалежне оцінювання зараз є головним інструментом оцінювання якості математичної підготовки для учнів старших класів України. Зокрема, воно використовується для проведення державної підсумкової атестації навчальних досягнень випускників, а також як інструмент для конкурсного відбору абітурієнтів до українських ЗВО. Таким чином, ми не сумніваємось у важливості та необхідності досліджень різних аспектів підготовки до ЗНО з математики. Одним із таких аспектів є тематичне повторення шкільного курсу математики.

**Матеріали і методи.** Для досягнення цієї мети ми застосовуємо кілька емпіричних методів: спостереження за навчальним процесом учнів під час їх навчання на курсах підготовки до ЗНО з математики та аналіз результатів їхніх досягнень. У дослідженні також використовувалася набір методів наукового пізнання: порівняльний аналіз для з'ясування різних поглядів на проблему та визначення напрямку дослідження; систематизація та узагальнення для формулювання висновків та рекомендацій; узагальнення авторського педагогічного досвіду та спостережень.

**Результати.** Виходячи з авторського досвіду систематизації та повторення шкільного курсу математики під час підготовки до ЗНО, ми пропонуємо розділити весь курс математики на 10 логічних змістових блоків. У цій статті ми надаємо тематичні тести до змістових блоків «Координати та вектори», «Елементи комбінаторики та стохастики», а також відповіді на них. Ми також вирішуємо деякі основні завдання цих тестів і даємо кілька методичних коментарів щодо цих розв'язань. Векторні та координатні методи дуже часто полегшують процес розв'язування геометричних задач порівняно з традиційними методами. Статистичні та ймовірнісні методи використовуються як засіб моделювання процесів і явищ реального світу, а тому їх вивчення сприяє формуванню світогляду дитини.

**Висновки.** Ми віримо, що добре організована тематична підготовка до ЗНО та ДФА з математики дозволить вчителям подолати проблеми, з якими стикаються учні при систематизації та повторенні шкільного курсу математики. Ця публікація завершує серію наших статей про сучасну тематичну підготовку до ЗНО з математики. У них ми окреслили своє бачення методології його організації, а також поділилися нашими дидактичними матеріалами та методичними порадами.

**Ключові слова:** ЗНО з математики, ДПА з математики, тематична підготовка, навчальні досягнення учнів, тематичні тести, основні завдання, координати, вектори, комбінаторика, стохастика.