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НЕЙТРОННЕ ОЦІНЮВАННЯ МАТЕМАТИЧНИХ УМІНЬ СТУДЕНТІВ

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АНОТАЦІЯ

Формулювання проблеми. *Оцінювання є важливою складовою навчального процесу, оскільки воно допомагає викладачеві визначити помилки студентів і покращити їх результати шляхом відповідної адаптації методів навчання. У даній роботі ми досліджуємо проблему оцінювання загальної успішності студента, коли викладач невпевнений у правильності виставлених оцінок. Це трапляється через те, що викладач не має достатньо часу, щоб належним чином оцінити математичні навички студентів, або тому, що у випадку письмового іспиту/тесту деякі студенти не чітко представили або не обґрунтували належним чином свої відповіді.*

Матеріали та методи. *Методи нечіткого оцінювання з використанням нейтронних множин і сірих чисел, а також розрахунок індексу середнього балу використовуються в цій роботі для оцінки середньої успішності студента в класі, якості успішності та загальної успішності, коли вчитель сумнівається щодо оцінки, виставленої деяким студентам.*

Результати. *Стаття зосереджена на застосуванні, розробленому для оцінювання математичних навичок студентів-інженерів двох факультетів Школи інженерії Вищого технологічного освітнього інституту Західної Греції (Університет Пелопоннесу), які перебувають на першому терміні навчання, за допомогою якісних (лінгвістичних) оцінок. Методика навчання на першому факультеті (експериментальна група) передбачала поєднання використання комп'ютера та аудиторних лекцій, тоді як на другому факультеті (контрольна група) проводилися лекції лише класичним методом.*

Висновки. *Використання нейтронних множин є корисним інструментом для оцінювання загальної успішності студента, коли викладач має сумніви щодо точності виставлених їм оцінок. Результати аудиторного застосування продемонстрували перевагу експериментальної групи. Ця перевага, однак, була значною щодо його середньої та загальної продуктивності (нейронна оцінка), але досить незначною щодо його якісних показників. Це є переконливим свідченням того, що використання комп'ютерів у навчальному процесі більше допомагає студентам з низьким та середнім рівнем навчальних досягнень.*

КЛЮЧОВІ СЛОВА: нечіткі методи оцінювання; сірі числа; індекс середнього балу; нейтронні множини; нейтронні трійки.

NEUTROSOPHIC ASSESSMENT OF STUDENT MATHEMATICAL SKILLS

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ABSTRACT

Formulation of the problem. *Assessment is an important component of the teaching process, because it helps the instructor to determine the student mistakes and to improve their performance by adapting suitably his/her teaching methods. In this work we investigate the problem of evaluating the student overall performance, when the teacher is not sure about the accuracy of the grades assigned to them. This happens, either because the teacher had not enough time to assess properly the students' mathematical skills, or because, in case of a written examination/test, some students did not present clearly or did not justified properly their answers.*

Materials and methods. *Fuzzy assessment methods using neutrosophic sets and grey numbers, as well as the calculation of the Grade Point Average (GPA) index are used in this work for the assessment of a student class mean performance, quality performance, and overall performance when the teacher has doubts about the grades assigned to some students.*

Results. *The paper focuses on a classroom application designed for the assessment, with qualitative (linguistic) grades, of mathematical skills of the engineering students of two Departments of the School of Engineering of the Graduate Technological Educational Institute (TEI) of Western Greece (University of Peloponnese) being at their first term of studies. The instructor was the same person for both Departments. The teaching methodology for the first Department (experimental group) involved a combined use of computers and classroom lectures, whereas for the second Department (control group) involved only lectures in the classical way on the board.*

Conclusions. *The use of neutrosophic sets provides a useful tool for evaluating the student overall performance when the teacher has doubts about the accuracy of the grades assigned to them. The outcomes of the classroom application demonstrated a superiority of the experimental group. This superiority, however, was significant with respect to its mean and overall performance (neutrosophic assessment), but rather negligible with respect to its quality performance. This gives a strong indication that the use of computers in the teaching process helps more the mediocre and weak students and not so much the good students.*

KEYWORDS: Fuzzy Assessment Methods; Grey Numbers (GNs); Grade Point Average (GPA) Index; Neutrosophic Sets (NSs); Neutrosophic Triplets.

INTRODUCTION

Assessment is an important component of the teaching process, because it helps the instructor to determine the student mistakes and to improve their performance by adapting suitably his/her teaching methods.

When numerical grades are used for the assessment process, then the student ***mean performance*** is evaluated by calculating the mean value of the grades assigned to them. Another, popular in many countries, method for evaluating the overall performance of a student group is the calculation of the ***Grade Point Average (GPA)*** index (Voskoglou, 2017, p. 125), which is calculated by the formula

$$\text{GPA} = \frac{0n_F + n_D + 2n_C + 3n_B + 4n_A}{n} \quad (1)$$

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In formula (1), n denotes the total number of students of the corresponding group, A=excellent, B=very good, C=good, D=mediocre and F=fail are the linguistic grades assigned to the students, and n_X denotes the number of students who obtained the grade X=A, B, C, D, F. In other words, the GPA index is a weighted average in which greater coefficients (weights) are assigned to the higher grades; therefore it assesses not the mean, but the ***quality performance*** of the student group

Note that, since in the worst case ($n=n_F$) is GPA=0 and in the ideal case ($n=n_A$) is GPA=4, we have in general that

$$0 \leq GPA \leq 4 \quad (2).$$

Example 1: Assume that in a written test the 20 in total students of a class obtained the following grades in the numerical climax 1-100: 98: 2 (students), 95: 2, 90: 3, 85:1, 80:2, 75: 4, 60:2, 50:2, 40:1, 30:1. Evaluate the mean and the quality performance of the class.

Solution: Following widely accepted standards, we assign numerical scores to the linguistic grades A, B, C, D, F.as follows: A → [85, 100], B → [75, 84], C → [60, 74], D → [50, 59], F → [49, 0].

The mean value of the student scores is equal to $\frac{2*98+2*95+3*90+85+2*80+4*75+2*60+2*50+40+30}{20}=74.55$, which shows that the class demonstrated a good (C) mean performance.

Also, by formula (1) we get that GPA= $\frac{2+2*2+3*6+4*8}{20}=2.8$. Thus, with respect to inequality (2), one finds that $\frac{2.8}{4}=0.7$, or 70%. This shows that the class demonstrated a good quality performance too.

In many cases, however, the assessment of the student performance is realized by using qualitative (linguistic) instead of numerical grades. In such cases, the assessment of the student mean performance cannot be realized by calculating the average of numerical scores. To solve this problem, we proposed in earlier works several methods using principles of fuzzy logic, the most important of which are reviewed in (Voskoglou, 2019a). In this work we use grey numbers for overcoming this problem.

Also note that frequently the teacher has doubts for the grades assigned to some students, either because he had not enough time to assess properly their performance, or, in case of written tests/examinations, because their answers were not very clear or well justified. In this paper we use neutrosophic sets as tools to obtain a reasonable criterion for evaluating the student overall performance in such cases.

The present work focuses on a classroom application for the assessment of mathematical skills of engineering students. The rest of the paper is formulated as follows: The next section contains the necessary information about fuzzy sets, neutrosophic sets and grey numbers, needed for the purposes of this work. The fuzzy methods that we use for the student assessment are developed in the third section and the classroom application is presented in the fourth section. The paper closes with the final conclusions and some hints for future research, which are included in its fifth and last section.

MATHEMATICAL BACKGROUND

Fuzzy Sets and Logic

The development of human science and civilization owes a lot to Aristotle's (384-322 BC) ***bivalent logic (BL)***, which was in the center of human reasoning for centuries. BL is based on the ***"Principle of the Excluded Middle"***, according to which each proposition is either true or false.

Opposite views, however, appeared also early in the human history supporting the existence of a third area between true and false, where these two notions can exist together; e.g. by Buddha Siddhartha Gautama (India, around 500 BC), by Plato (427-377 BC), more recently by the Marxist philosophers, etc. Integrated propositions of multi-valued logics reported, however, only during the early 1900's, mainly by Lukasiewicz and Tarski (Voskoglou, 2019a, Section 2). According to the Lukasiewicz's ***"Principle of Valence"*** propositions are not only either true or false, but they may have intermediate truth-values too.

Zadeh, replacing the characteristic function of a crisp subset of the universe U with the ***membership function*** m: U→ [0, 1], introduced the concept of ***fuzzy set (FS)*** (Zadeh, 1965), in which each element x of U has a ***membership degree*** m(x) in the unit interval. The closer m(x) to 1, the better x satisfies the characteristic property of the corresponding FS. For example, if A is the FS of the tall men of a country and m(x) = 0.8, then x is a rather tall man. On the contrary, if m(x) = 0.4, then x is a rather short man. Formally, a FS A in U can be written as a set of ordered pairs in the form

$$F = \{(x, m(x)): x \in U\} \quad (3)$$

Zadeh also introduced, with the help of FS, the infinite-valued in the unit interval ***fuzzy logic (FL)*** (Zadeh, 1973), on the purpose of dealing with the existing in the everyday life partial truths. FL, in which truth values are modelled by numbers in the unit interval, embodies the Lukasiewicz's "Principle of Valence".

Uncertainty can be defined as the shortage of precise knowledge or complete information on the data that describe the state of a situation. It was only in a second moment that FS theory and FL were used to embrace uncertainty modelling. This happened when membership functions were reinterpreted as ***possibility*** distributions (Zadeh, 1978, Dubois & Prade, 2001). Zadeh (1978), articulated the relationship between possibility and ***probability***, noticing that what is probable must preliminarily be possible.

Probability theory used to be for many years the unique tool in hands of the specialists for dealing with problems connected to uncertainty. Probability, however, was proved to be suitable only for tackling the cases of uncertainty which are due to ***randomness*** (Kosko, 1990). Randomness characterizes events with known outcomes which, however, cannot be predicted in advance, e.g. the games of chance. FSs, apart from randomness, tackle also successfully the uncertainty due to ***vagueness***, which is created when one is unable to distinguish between two properties, such as "a good player" and "a mediocre player". For general facts on FSs and the connected to them uncertainty we refer to the book (Klir & Folger, 1988).

Neutrosophic Sets

Several generalizations and extensions of the theory of FSs have been developed during the last years for the purpose of tackling more effectively all the forms of the existing in real world uncertainty. The most important among them are briefly reviewed in (Voskoglou, 2019b).

Atanassov (1986), considered, in addition to Zadeh's membership degree, the degree of ***non-membership*** and extended FS to the notion of ***intuitionistic FS (IFS)***. Smarandache (1998), inspired by the frequently appearing in real life neutralities - like <friend, neutral, enemy>, <win, draw, defeat>, <high, medium, short>, etc. - generalized IFS to the concept of ***neutrosophic set (NS)*** by adding the degree of ***indeterminacy* or *neutrality***. The word "neutrosophy" is a synthesis of the word "neutral΄ and the Greek word "sophia" (wisdom) and means "the knowledge of the neutral thought". The simplest form of a NS is defined as follows:

Definition 1: A ***single valued NS (SVNS)*** A in the universe U is of the form

$$A = \{(x,T(x),I(x),F(x)): x\in U,\ T(x),I(x),F(x)\in[0,1],\ 0 \le T(x) + I(x) + F(x) \le 3\} \quad (4)$$

In equation (4) T(x), I(x), F(x) are the degrees of ***truth*** (or membership), indeterminacy (or neutrality) and ***falsity*** (or non-membership) of x in A respectively, called the ***neutrosophic components*** of x. For simplicity, we write A<T, I, F>. Indeterminacy is defined to be in general everything that exists between the opposites of truth and falsity (Smarandache, 2021).

Example 2: Let U be the set of the players of a soccer club and let A be the SVNS of the good players of the club. Then each player x is characterized by a ***neutrosophic triplet*** (t, i, f) with respect to A, with t, i, f in [0, 1]. For example, x(0.7, 0.1, 0.4) ∈ A means that there exists a 70% belief that x is a good player, but at the same time there exist a 10% doubt about it and a 40% belief that x is not a good player. In particular, x (0, 1, 0) ∈ A means that we do not know absolutely nothing about the quality of player x (new player).

If the sum T(x) + I(x) + F(x) < 1, then it leaves room for ***incomplete information*** about x, if it is equal to 1 for ***complete information*** and if it is >1 for ***inconsistent*** (i.e. contradiction tolerant) ***information*** about x. A SVNS may contain simultaneously elements leaving room to all the previous types of information. All notions and operations defined on FSs are naturally extended to SVNSs (Wang et al., 2010).

Summation of neutrosophic triplets is equivalent to the union of NSs. That is why the neutrosophic summation and implicitly its extension to neutrosophic scalar multiplication can be defined in many ways, equivalently to the known in the literature neutrosophic union operators (Smarandache, 2016). For the needs of the present work, writing the elements of a SVNS A in the form of neutrosophic triplets and considering them simply as <u>ordered triplets</u> we define addition and scalar product as follows:

Definition 2: Let (t_1, i_1, f_1), (t_2, i_2, f_2) be in A and let k be a positive number. Then:

- The sum $(t_1, i_1, f_1) + (t_2, i_2, f_2) = (t_1+ t_2,\ i_1+ i_2, f_1+ f_2)$ (5)
- The scalar product $k(t_1, i_1, f_1) = (kt_1, k i_1, kf_1)$ (6)

Remark 1: Summation and scalar product of the elements of a SVNS A with respect to Definition 2 <u>need not</u> be closed operations in A, since it may happen that $(t_1+ t_2)+(i_1+ i_2)+(f_1+ f_2)>3$ or $kt_1+k i_1 + kf_1>3$. With the help of Definition 2, however, one can define <u>in A</u> the *mean value* of a finite number of elements of A as follows:

Definition 3: Let A be a SVNS and let (t_1, i_1, f_1), (t_2, i_2, f_2), …., (t_k, i_k, f_k) be a finite number of elements of A. Assume that (t_i, i_i, f_i) appears n_i times in an application, i = 1,2,…., k. Set $n = n_1+n_2+….+n_k$. Then the ***mean value*** of all these elements of A is defined to be the element (t_m,i_m,f_m) of A calculated by

$$\frac{1}{n}[n_1(t_1, i_1,f_1)+n_2(t_2,i_2,f_2)+….+n_k(t_k,i_k, f_k)] \quad (7)$$

Grey Numbers

The theory of ***grey systems*** (Deng, 1982) introduces an alternative way for managing the uncertainty in case of approximate data. A grey system is understood to be any system which lacks information, such as structure message, operation mechanism or/and behavior document.

Closed real intervals are used for performing the necessary calculations in grey systems. In fact, a closed real interval [x, y] could be considered as representing a real number T, termed as a ***grey number (GN)***, whose exact value in [x, y] is unknown. We write then T ∈ [x, y]. A GN T, however, is frequently accompanied by a ***whitenization function*** f: [x, y] → [0, 1], such that, if f(a) approaches 1, then a in [x, y] approaches the unknown value of T. If no whitenization function is defined, it is logical to consider as a representative crisp approximation of the GN T the real number

$$V(T) = \frac{x+y}{2} \quad (8)$$

The arithmetic operations on GNs are introduced with the help of the known arithmetic of the real intervals (Moore et al., 1995). In this work we are going to make use only of the addition of GNs and of the scalar multiplication of a GN with a positive number, which are defined as follows:

Definition 4: Let $A \in [x_1, y_1]$, $B \in [x_2, y_2]$ be two GNs and let k be a positive number. Then:

- The sum: A+B is the GN $A+B \in [x_1+y_1, x_2+y_2]$ (9)
- The scalar product kA is the GN $kA \in [kx_1, ky_1]$ (10)

FUZZY ASSESSMENT METHODS WITH QUALITATIVE GRADES

Mean Performance

As said before, when using qualitative grades for the assessment the mean performance of a student group cannot be evaluated with the classical method of calculating the mean value of the student scores. For overcoming this difficulty, we assign here to each qualitative grade a GN, denoted for simplicity with the same letter, as follows: A = [85, 100], B = [75, 84], C = [60, 74], D = [50, 59], F = [0, 49]. The choice of the above GNs, although it corresponds to generally accepted standards, is not unique. For example, for a more strict assessment, one may choose A= [90, 100], B = [80, 89], C= [70, 79], D = [60, 69], F = [0, 59], or otherwise. Such changes, however, does not affect the generality of our method

Assume now that, from the n in total students of the group, n_X obtained the grade X=A, B, C, D, F. It is logical then to accept that the crisp approximation V(M) of the GN

$$M = \frac{1}{n}(n_A A + n_B B + n_C C + n_D D + n_F F) \quad (11)$$

can be used for estimating the mean performance of the student group.

Neutrosophic Assessment

When the teacher has doubts about the grades assigned to some students, the most suitable method for assessing the overall performance of a student group is to use NSs as tools. In our case, considering the NS of the good students of the group, we introduce neutrosophic triplets characterizing the individual performance of each student and we calculate the mean value of all these triplets with the help of equation (7) for obtaining the proper conclusions about the group's overall performance. In order to have complete information for each student's performance, the sum of the component of each triplet must be equal to 1.

THE CLASSROOM APPLICATION

The target of the following classroom application was the assessment of mathematical skills of engineering students. The subjects were the first term students of two departments of the School of Engineering of the Graduate TEI of Western Greece (60 students in each department) during the course "Higher Mathematics I", which includes Complex Numbers, Differential and Integral Calculus in one variable and elements from Linear Algebra. According to the grades obtained in the PanHellenic examination for entrance in Higher Education, the potential of the two departments in mathematics was about the same. The course's instructor was also the same person, but the teaching methods followed were different. Namely, the teaching methodology for the first department (experimental group) involved a combination of classroom lectures and computer applications using the proper mathematical software, whereas the classical method with lectures on the board was applied for the second department (control group).

The results of the common final examination, after the end of the course, were the following:

- Department I: A: 9 students, B: 15, C: 18, D: 12, F: 6
- Department II: A: 12, B: 15, C: 9, D: 12, F: 12

Applying the previously described assessment methods , we evaluated the performance of the two departments as follows:

Mean performance

By equation (11) one finds that

$$M_I = \frac{1}{60}\ (9[85,100]+15[75,84]+18[60,74]+12[50,59]\ +6[0,49]) = \frac{1}{60}\ [3570,4994] \approx [59.5,83.23].$$

Therefore, equation (8) gives that $V(M_I) \approx 71.36$, which shows that the experimental group demonstrated a good (C) mean performance. In the same way one finds that $V(M_{II}) \approx 62.56$, which shows that the control group also demonstrated a good mean performance, which, however, was 8.8% worse than that of the experimental group.

Quality performance

Equation (1) gives that $GPA_I = \frac{12+2*18+3*15+4*9}{60}$ = 2.12 and similarly GPA_{II}=2.05, which shows that the experimental group demonstrated a slightly better quality performance. In fact, with the help of equation (2) it is easy to check that the superiority of the experimental group in this case is only 0.07*25=1.75%.

Remark 2: When two groups have the same GPA index, the calculation of it is not sufficient to show which of them performed better. In such cases the ***Rectangular Fuzzy Assessment Model (RFAM),*** which is based on the ***Center of Gravity (COG) defuzzification technique*** can be used to overcome this difficulty (Voskoglou, 2017, pp. 126-130).

Neutrosophic Assessment

Some of the student answers in the final examination were not clearly presented or well justified. As a result, the instructor was not quite sure for the accuracy of the grades assigned to them. For this reason, we decided to apply the neutrosophic method too for the assessment of the two departments' overall performance. For this, starting from the students with the higher grades, let us denote by S_i, i=1,2,….,60, the students of each department. Considering the NS of the good students, we assigned neutrosophic triplets to all students of the two departments as follows:

- Department I: S_1-S_{32}: (1,0,0), S_{33}-S_{38}: (0.8,0.1,0.1), S_{39}-S_{42}: (0.7,0.2,0.1), S_{43}-S_{46}: (0.4,0.2,0.4), S_{47}-S_{50}: (0.3,0.2,0.5), S_{51}-S_{53}: (0.2,0.2,0.6), S_{54}-S_{55}: (0.1,0.2,0.7), S_{56}-S_{57}: (0,0.2,0.0.8), , S_{578}-S_{60}: (0,0,1).
- Department II: S_1-S_{31}: (1,0,0), S_{32}-S_{35}: (0.8,0.1,0.1), S_{36}: (0.7,0.1,0.2), S_{35}-S_{43}: (0.4,0.1,0.5), S_{44}-S_{46}: (0.3,0.2,0.5), S_{47}-S_{50}: (0.2,0.2,0.6), S_{51}-S_{52}: (0.1,0.2,0.7), S_{53}-S_{58}: (0,0.3,.0.7), , S_{59}-S_{60}: (0,0,1).

Then, by equation (7), the mean value of the neutrosophic triplets of Department I is equal to

$\frac{1}{60}$ [32(1,0,0)+6(0.8,0.1,0.1)+4(0.7,0.2,0.1)+4(0.4,0.2,0.4)+4(0.3,0.2,0.5)+3(0.2,0.2,0.6)+2(0.1,0.2,0.7)+2(0,0.2,0.0.8)+3(0,0,1)≈ ≈(0.72, 0.07, 0.21). In the same way we found that the mean value of the neutrosophic triplets of Department II is equal to (0.65, 0.08, 0.27). Thus, the probability for a random student of Department I to be a good student is 72%, but at the same time there exists a 7% doubt about it and a 21% probability to be not a good student. Also, the probability for a random student of Department II to be a good student is 65%, with a 8% doubt about it and a 27% probability to be not a good student. Consequently, the experimental group, despites the doubts of the instructor for the accuracy of the grades assigned to the students, demonstrated a better overall performance.

CONCLUSION

The classroom application presented in this work demonstrated a superiority of the experimental group with respect to the control group. This superiority, however, was significant concerning the two groups' mean and overall (in terms of the neutrosophic method) performance, but rather negligible concerning their quality performance. This provides a strong indication that the use of computers in the teaching process benefits more the mediocre and the weak in mathematics students, but less the good students. Much more experimental research is needed, however, for obtaining safer conclusions

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