ФІЗИКО-МАТЕМАТИЧНА ОСВІТА
(PHYSICAL AND MATHEMATICAL EDUCATION)
p-ISSN: 2413-1571, e-ISSN: 2413-158X 2025, 40(3), https://doi.org/10.31110/fmo2025.v40i3-06
UDC: 372.851

ПОСТАНОВКА ТА РОЗВ'ЯЗАННЯ ДОСЛІДНИЦЬКИХ ЗАДАЧ ПРИ ВИКЛАДАННІ МАТЕМАТИЧНИХ ДИСЦИПЛІН

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АНОТАЦІЯ

Формулювання проблеми. *Дослідницькі задачі є необхідною складовою освітнього процесу, їх використання сприяє розвитку критичного та креативного мислення, формуванню вміння діяти в нестандартній ситуації. Поряд з необхідністю ґрунтовних педагогічних досліджень щодо теоретичних основ впровадження технології навчання через дослідження, виникає ціла низка практичних питань, пов'язаних з підготовкою викладача до організації дослідницької діяльності в освітньому процесі. Мета цієї статті – зробити огляд наявних досліджень щодо дослідницького підходу до навчання, описати власний досвід постановки дослідницьких задач при викладанні математичних дисциплін і організації їх розв'язання студентами, проаналізувати можливості знаходження джерел дослідницьких задач та самостійного їх створення.*

Матеріали і методи. *Використона: аналіз застосування окремих компонентів теорії розв'язування дослідницьких задач, аналіз наявних досліджень впливу результатів дослідницької діяльності на підвищення мотивації студентів. У дослідженні використовувались підручники з вищої математики, алгебри та теорії чисел, основ математики для закладів вищої освіти, опрацьовувались наукові публікації та їх обговорення.*

Результати. *Продемонстровано можливості організації дослідницької діяльності студентів при викладанні математичних курсів, використання теорії розв'язання дослідницьких задач, проаналізовано окремі можливості створення дослідницьких задач, розглянуто методи розв'язання запропонованих дослідницьких задач (метод фокального об'єкта, акумулювання навчального матеріалу, брейнштурму).*

Висновки. *Математичні курси є сприятливою базою для забезпечення дослідницької діяльності студентів університету. В статті акцентується увага на включенні дослідницьких задач в процес навчання на постійній основі. Викладачам університетів слід удосконалювати свій власний досвід організації дослідницької діяльності студентів, а також досвід викладачів зарубіжних університетів, які вже певний час працюють над реалізацією технології навчання через дослідження. Зрозуміло, що перехід до цієї технології буде тривалим і потребуватиме поступових змін в навчальних програмах спеціальностей. Необхідними будуть також дослідження, пов'язані з методиками підвищення мотивації студентів для забезпечення результативності дослідницької діяльності.*

КЛЮЧОВІ СЛОВА: дослідницька діяльність; дослідницька задача; освітній процес; неперервне мислення; технології навчання; методи дослідження; проєктно-дослідницька діяльність; методи пізнання.

SETTING AND SOLVING RESEARCH PROBLEMS IN TEACHING MATHEMATICAL DISCIPLINES

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ABSTRACT

Formulation of the problem. *Research tasks are a necessary component of the educational process, and their use contributes to the development of critical and creative thinking, and the formation of the ability to act in a non-standard situation. Along with the need for thorough pedagogical research on the theoretical foundations of the implementation of research-based learning technology, a number of practical issues arise related to the preparation of teachers for the organization of research activities in the educational process. The purpose of this article is to review the available research on the research approach to teaching, to describe my own experience of setting research tasks in teaching mathematics and organizing their solution by students, to analyze the possibilities of finding sources of research tasks and creating them independently.*

Materials and methods. *Methods used: analysis of the application of individual components of the theory of research problem solving, analysis of existing studies on the impact of research results on increasing student motivation. The study used textbooks on higher mathematics, algebra and number theory, and the basics of mathematics for higher education institutions, as well as scientific publications and their discussions..*

Results. *The article demonstrates the possibilities of organizing students' research activities in teaching mathematical courses, using the theory of solving research problems, analyzes some possibilities of creating research problems, and considers methods of solving the proposed research problems (the method of focal object, accumulation of educational material, brainstorming).*

Conclusions. *Mathematical courses are a favorable basis for ensuring research activities of university students. The article emphasizes the inclusion of research tasks into the learning process on a regular basis. University professors should improve their own experience in organizing students' research activities, as well as the experience of professors from foreign universities who have been working on the implementation of research-based learning for some time. It is clear that the transition to this technology will be long and will require gradual changes in the curricula of the specialties. Research will also be needed on methods of increasing student motivation to ensure the effectiveness of research activities.*

KEYWORDS: research activity; research task; educational process; continuous thinking; learning technologies; research methods; project research activity; methods of cognition.

ДЛЯ ЦИТУВАННЯ: Stegantsev E., Chernienko A. Setting and Solving Research Problems in Teaching Mathematical Disciplines. *Фізико-математична освіта*, 2025. Том 40. № 3. С. 37-43. https://doi.org/10.31110/fmo2025.v40i3-06.

FOR CITATION: Stegantsev, E., & Chernienko, A. (2025). Setting and Solving Research Problems in Teaching Mathematical Disciplines. *Physical and Mathematical Education*, *40*(3), 37-43. https://doi.org/10.31110/fmo2025.v40i3-06.

INTRODUCTION

Setting and solving research problems contributes to the development of science and is a means of attracting young researchers to new scientific fields. That is why solving research problems is an important part of teaching mathematics. The introduction of research-based learning into education is specified by the National Doctrine of Education Development of Ukraine in the XXI century and is being thoroughly studied by domestic and foreign scientists (Bulvinska, 2019; Kozak, 2016; Vorozhbit-Gorbatyuk et al.,2021).

In the context of research-based learning technology at the university, students are ideally involved in research conducted by teachers. This understanding of the role of the teacher differs significantly from the vision of the teacher as an experienced researcher who organizes students' projects and research activities or uses problem-based learning technology in the educational process. It is clear that today, a large number of teachers are not ready for the full implementation of research-based learning technology, and this is a task for the future that should be gradually and continuously solved.

Unlike typical tasks, the solution of which is often algorithmic, research tasks require the use of general and special methods of cognition. Of particular importance are tasks that develop such important mental skills as the ability to generalize, identify individual cases, reason by analogy, find similarities and differences, etc. The peculiarity of research problems is the fact that a broad mathematical outlook, ingenuity, and creativity are important for solving them, but, on the other hand, it is precisely in solving research problems that these qualities are formed.

The important tasks of higher education are to develop such professional competences of graduates as creativity and flexibility of thinking, the ability to make decisions in non-standard situations, as well as the formation and development of research competence of higher education students, especially of mathematical specialities. When engaging students majoring in secondary education in research, it is also desirable to develop their ability to organize and effectively manage students' research activities (Babak & Vorozhbit-Gorbatyuk, 2021; Proshkin et al., 2018).

Related to the idea of developing research skills in students is the idea of the need to develop so-called non-linear thinking as a new style of scientific thinking. Thinking in the process of invention, for example, is non-linear, as the intended objects, knowledge, and end results are initially unknown and gradually determined by self-organized personal and collaborative efforts. One of the first works on non-linear thinking was the book (Dobronravova, 1990). This idea was backed by numerous domestic and foreign scientists (Harkki et al., 2021; Kremen, 2005).

When solving research problems, it is necessary to think not only about the relationship between specific concepts but also between different sections of mathematics and other disciplines (STEM education). The students (and teachers) need to learn to cope with uncertainty, improvise, and be prepared to analyze the results, which may not be initially foreseeable. The issues of organizing research activities at different stages of the educational process are discussed at conferences, on the pages of scientific and popular science journals, where you can find recommendations on effective approaches to developing advanced skills and thinking necessary to overcome the complex challenges of the modern world (Ammar et al., 2024; Proshkin & Proshkina, 2016).

Realizing the importance of this issue, proactive teachers share their experience of organizing active learning through clubs, elective courses, and supervising the writing of papers in the Junior Academy of Sciences. Famous scientists take part in various summer schools. The organization of research activities provides a certain degree of convergence between university mathematics and school mathematics. The hypothesis that active learning improves performance in science, engineering, and mathematics is confirmed in the work (Freeman et al., 2014). To do this, 225 studies of student performance in undergraduate science, technology, engineering, and mathematics (STEM) courses in traditional lecture versus active learning were analyzed. The results raise questions about the continued use of traditional lectures and support active learning as a desirable, empirically supported teaching practice. When organizing research activities for pupils or students, a good result can only be expected if the proposed research tasks are interesting for them and can motivate students to solve them. Stimulating student motivation is the key issue for achieving the planned learning outcomes. A good course design is essential to keep students motivated to solve the posed tasks. When using problem-based learning technology, students' independence and the ability to function effectively in teams are important. Both are considered at work (Nor Farida et al., 2012) as the motivating elements. The work (Hrybiuk, 2022) describes the results of an experimental study confirming that the use of computer-oriented methodological systems of natural and mathematical research training provides optimal concentration of educational resources, focus of content, and technologies for preparing students for research work, an increase in motivation and efficiency of students' learning.

Aim of the study. The most obvious examples of research activities of university students are writing term papers and qualification papers. But very often this work is sporadic, limited in time, and therefore insufficient to develop research skills. Of course, it is important to ensure that students are constantly involved in research, to offer research tasks during the study of disciplines, starting from the first year. This is the most effective way to ensure quality teaching of natural and mathematical disciplines at all levels of the educational process.

The purpose of this article is to describe own experience of setting research tasks in teaching mathematical disciplines and organizing their solution by students, to analyze the possibilities of finding sources of research tasks and creating them independently.

RESEARCH METHODS

The demonstration of the possibilities of creating research ideas in teaching mathematics courses, analysis of the application of individual components of the theory of research problem solving, application of the method of accumulation of the required amount of educational material, the method of focal objects, the structural approach to research problems, involving students in research discussions, analysis of existing studies on the impact of research results on increasing student motivation and vice versa.

RESEARCH RESULTS

Setting the research problems under the study of axiomatic theories

In studying the fundamentals of geometry, the students are introduced to various axiomatic theories of the mathematical structure of Euclidean geometry. The theory based on Gilbert's system of axioms is thoroughly considered. Acquaintance with other systems of axioms of Euclidean geometry (Weyl, Aleksandrov, Kolmogorov) is important for mastering the general issues of the axiomatic method, in particular, for developing skills in proving the consistency, independence, and completeness of axiom systems. It is of considerable didactic and methodological importance to prove the equivalence of different axiomatic theories of Euclidean geometry, for example, the Gilbert and Weyl theories. For the deep understanding of this equivalence, it is important to be able to "find the place" of the particular statement in each axiomatic theory, i.e., to build the sequence of statements whose consequence is this statement. This kind of task can certainly be attributed to research.

For example, let us consider the triangle exterior angle theorem: "the exterior angle of a triangle is greater than every interior angle that is not adjacent to it". This theorem plays a very important role in Gilbert's axiomatic theory. It is the consequence of the first three groups of the axiom system and is used to prove many theorems of absolute geometry (Gilbert, 1948). The question of its place in Weyl's axiomatic theory is quite natural when studying the basics of geometry.

The organization of the study of this issue can be characterized as the implementation of the process of project research activity (Grib'yuk & Yunchik, 2016), because it included the following stages: selection of the problem (due to its importance in Gilbert's axiomatic theory); selection of the search concept, collection and analysis of data (by accumulating the necessary amount of educational material); transformation of ideas into the structure (taking into account the presence of the structure of the partially ordered set in the set of statements of the axiomatic theory).

Here is the sequence of statements obtained as the result of the study, each of which is the consequence of the previous ones and the consequence of which is the theorem on the external angle of a triangle in the axiomatic Weyl theory.

Statement 1. In any triangle ABC the following relation takes place

$$AB - BC < AC < AB + BC.$$

Statement 2. In any triangle ABC the following relation takes place:

$$1)\ BC = BA\cos B + CA\cos C;$$
$$2)\ CA = CB\cos C + AB\cos A;$$
$$3)\ AB = AC\cos A + BC\cos B.$$

Statement 3. The following inequalities are fulfilled for internal angles in any triangle ABC :

$$1)\ \cos A + \cos B + \cos C > 1;$$
$$2)\ -\cos A + \cos B + \cos C > -1.$$

Statement 4. The sum of the cosines of the two interior angles of any triangle is positive.

Statement 5 (on the exterior angle of the triangle). The exterior angle of a triangle is greater than every interior angle of the triangle that is not adjacent to it.

Proof. Let us demonstrate that the exterior angle α at vertex *C* of the triangle ABC is greater than angle B. According to statement 4, for the interior angles B and C we obtain $\cos B > -\cos C = \cos(\pi - C) = \cos\alpha$, and consequently $\angle B < \alpha$.

Similar questions can be asked about the triangle angle sum theorem, the Pythagorean theorem, etc.

Examples of the research tasks in the course of algebra and number theory

Let us consider ***the application of the properties of the reciprocal polynomial for the solution of a special type of third-degree equation.***

The concept of the polynomial plays an important role in algebra. Among the various types of polynomials, the irreducible polynomial over a field or ring holds a special place due to its possible applications, in particular, in mathematics and cryptography. Another interesting concept in polynomial theory is the concept of the reciprocal polynomial. This concept is not so widespread, but it also has applications, for example, in communication theory. Finding some effective combinations of the properties of the above types of polynomials can be considered as the research task.

For example, it is known that the solution of the equation

$$ax^3 + bx^2 + c = 0 \tag{1}$$

uses the Cardano method. In order to apply Cardano formulas directly, equation (1) is reduced to the following form

$$y^3 + my + n = 0, \tag{2}$$

using the sequence of the substitutions $m = \frac{b}{a}$, $n = \frac{c}{a}$ and $x = y - \frac{m}{3}$.

Let us consider the research task to find the method of reducing equation (1) to the standard form (2) in a different way, the purpose of which is to familiarize students with the concept of a reciprocal polynomial and its possible applications. Let us recall that the reciprocal polynomial $f^*(x)$ to the polynomial $f(x)$ can be found with the help of the formula $f^*(x)=x^n f\left(\frac{1}{x}\right)$ where n – the power of the original polynomial. We use the focal object method, which consists in focusing on the properties of the given object, comparing them with the properties of other objects (possibly randomly selected), formulating original ideas and implementing them (note that this scheme often includes other methods, such as the brainstorming method with the qualitative system of questions and analysis of student responses).

The application of these methods led to the following research results. The construction of the reciprocal polynomial for the polynomial in equation (1) immediately transforms this equation to the form (2), without applying the above substitutions. In order to write down the roots of the original equation, we use the property of reciprocal polynomials: the roots of the reciprocal polynomial are inverse to the roots of the original polynomial (Van-der-Varden, 1976). Thus, for the polynomial on the left-hand side of the equation $x^3+3x^2-20=0$ the reciprocal polynomial $f^*(x)$ has the form $f^*(x)=x^3\left(\frac{1}{x^3}+3\frac{1}{x^2}-20\right)=-20x^3+3x+1$. Hence, we obtain the equation $x^3-\frac{3}{20}x-\frac{1}{20}=0$, which has the standard form. Let us find the roots using Cardano formulas $x_1^*=\frac{1}{2}$, $x_2^*=-\frac{1}{4}+\frac{\sqrt{15}}{20}i$, $x_3^*-\frac{1}{4}-\frac{\sqrt{15}}{20}i$. Hence $x_1=2$, $x_2=-\frac{5}{2}-\frac{\sqrt{15}}{2}i$, $x_3=-\frac{5}{2}+\frac{\sqrt{15}}{2}i$ - are the roots of the original equation.

Let us consider ***the application of the properties of the reciprocal and irreducible polynomials in the preparation of the polynomial for the irreducibility test with the help of the Eisenstein criterion.***

This example confirms the fact that research activity can be made more than episodic, demonstrating to students how the results already obtained form the basis for formulating new research ideas, creating conditions for the implementation of stereotype removal operators as one of the components of the theory of research problem solving (Grib'yuk & Yunchik, 2016).

In order to test the polynomial for irreducibility over the field of rational numbers, the Eisenstein criterion is used. Sometimes, to apply this criterion, the polynomial must be transformed by moving to another variable, for example, using the Horner scheme.

Another way of preparing the polynomial for the application of the Eisenstein criterion arises from the following property of reciprocal polynomials: the polynomial, reciprocal to an irreducible polynomial, is irreducible (Markov & Razmyislovich, 1999). For example, in order to prove the irreducibility of the polynomial, let us find the polynomial that is reciprocal to the original polynomial. We obtain the polynomial $f^*(x)=x^3\left(\frac{7}{x^3}+\frac{14}{x^2}+1\right)=x^3+14x+7$, for which the application of the Eisenstein criterion leads to the conclusion that it is irreducible over the field of rational numbers.

Let us consider ***the study of the concepts of continued fractions and the Stern-Brocot tree.***

The continued fraction is the expression $q_0+\cfrac{1}{q_1+\cfrac{1}{q_2+\cfrac{1}{q_3+\ldots}}}$, where q_0 – is an integer, $q_1,q_2,q_3,\ldots$ – are natural numbers (coefficients of the fraction).

Let us note that the continued fraction, which corresponds to a rational number, is always finite. The following symbols are used to designate it

$$q_0+\cfrac{1}{q_1+\cfrac{1}{\ldots+\cfrac{1}{q_k}}}=<q_0,q_1,\ldots,q_k> \text{ or } n=<q_0,q_1,\ldots q_k-1,1>=q_0+\cfrac{1}{q_1+\cfrac{1}{\ldots+\cfrac{1}{q_k-1+\cfrac{1}{1}}}}.$$

The height of the positive rational number n is the sum of the coefficients of the continued fraction corresponding to that number, i.e. for $n=<q_0,q_1,\ldots,q_k>$ the height is $h(n)=q_0+q_1+\ldots+q_k$.

Stern-Brocot tree (or Pharey tree) is the way of arranging all non-negative irreducible ordinary fractions at the vertices of the ordered infinite root tree (Fig.1).

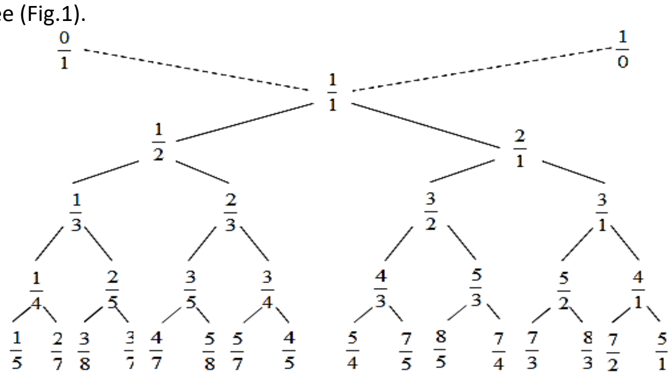

Fig. 1. Stern-Brocot tree

Джерело: авторська розробка. (АНГЛ.)

The layer in the Stern-Brocot tree is the set of fractions that are situated at the same distance from its root (fraction $\frac{1}{1}$). The tree contains all rational numbers, and each number occurs only once.

For example, let us consider the first two numbers of the fourth layer of the Stern-Brocot tree: $\frac{2}{5} = <0,2,2> = <0,2,1,1> 0 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1}}}$.

For each of these numbers, the height is equal to the number of layers, that is four. This fact is true for any layer. The corresponding theorem is formulated in [Lisitsyina et al., 2006], where the reader is invited to prove it. This is the situation for formulating the research problem.

Using the method of brainstorming in the process of studying Figure 1, we achieve fast (generative) thinking of students, we propose to formulate the theorem from (Lisitsyina et al., 2006) in the different form (note that the reformulation of the task is often used, helps to remove stereotypes, find ideas for solving the research problem). Here is this different formulation.

Theorem. The sets of rational numbers of height k and numbers, which belong to the k-th layer of the Stern-Brocot tree are equal.

Proving it requires some preparation. We will use the special representation of the Stern-Brocot node in the tree: we write the symbol L if we are moving down and left from the root $\frac{1}{1}$ of the tree to the given node, and the symbol R if we are moving down and right. For example, the number $\frac{3}{5}$ corresponds to the code LRL. In (Van-der-Varden, 1976) the symbols L and R correspond to the matrices $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ respectively. This allows us to reduce the process of converting the node of the Stern-Brocot tree into the regular fraction to multiplying second-order square matrices. In addition, you can identify the ancestors of this fraction and the layer of the Stern-Brocot tree in which the desired fraction is situated. For example, let us find the fraction, which corresponds to the symbol L^2RL (equivalent notation – $LLRL$). Four steps were taken to reach the desired node, so it belongs to the fifth layer. In matrix form, we have $L^2RL = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$. The ancestors of the desired fraction are fractions $\frac{1}{3}$ and $\frac{2}{5}$. According to the Stern-Brocot tree rule, the desired fraction is $\frac{3}{8}$.

Lemma. The $(k+1)$-th layer of the Stern-Brocot tree contains exactly 2^k nodes whose code sequence is L^k, $L^{k-1}R$,..., R^k.

For example, for $k=3$ we have the following code sequence: L^3, L^2R, LRL, LRR, RLL, RLR, RRL, RRR.

The proof of the theorem consists of two parts:

- The proof of the fact that each node of $(k+1)$-th layer of Stern-Brocot tree is a rational number of the height $(k+1)$;
- The proof of the correspondence between each rational number of the height $(k+1)$ and certain node of $(k+1)$-th layer of Stern-Brocot tree.

Sources of research tasks. Creating research tasks on your own

A good specialist not only solves other people's problems, but also knows how to set and solve his or her own. The ability to set goals indicates a deep understanding of the situation.

Generalization can be considered as the technique of the obtaining the new problems, including research ones.

Example. The problem "Find the last digit of the sum $1+2!+3!+4!+...+2012!$" can be generalized as follows:

- Find the last digit of the sum $1+2!+3!+4!+...+n!$ (technique of the parameter input);
- Solve the equation $1+2!+3!+4!+...+x! = y^2$ in natural numbers (the technique of rephrasing the condition).

Example. The generalization of the algorithm for finding the square root of a natural number (without the use of electronic tools and tables) to the case of the root of any natural power (Artemenko & Styeganceva, 2020).

Remark. One of the main principles of creating such systems of tasks is to gradually complicate the task at each stage of developing research skills. But there is another side to the generalization method. It is useful for finding ideas for solving certain problems. For example, to solve the task "Find the value of the expression $\sqrt{2022 \cdot 2023 \cdot 2024 \cdot 2025 + 1}$" the idea of moving to generalization "Find the value of the expression $\sqrt{x \cdot (x+1) \cdot (x+2) \cdot (x+3) + 1}$" is productive for finding a way to solve the posed problem.

Reading of the corresponding literature

For example, scientific, review, and popular articles. Such articles often formulate unsolved problems, possible generalizations of already solved problems, and questions of interest to the author. In addition, the reader may have his or her own list of questions, such as "what if...?", "is the opposite statement true?", "can the problem be generalized?", may want to reformulate the problem, consider the similar problem under different initial conditions, etc. Let us give two examples.

Example. Recently, R-functions introduced by V.L. Rvachev in 1963 have been widely used in various applications (Rvachev & Sheiko, 1995). They allow solving quite simply the inverse problem of analytic geometry - to write down the equation of the curve that bounds the given region. This turns out to be useful in solving problems of mathematical programming, optimal arrangement of objects in the plane, stability theory, mathematical physics, and others.

In the literature, the definition of R - function and examples of R - functions are given, but there is no example of a function that is not R -function. In (Velichko & Stegantseva, 2010), an example of the particular function is given, and it is proved that it cannot be R -function.

Example. In the course of differential geometry, the curvature and torsion of the spatial curve are studied. In the case of the curve belonging to the sphere (spherical curve), there is a certain relationship between curvature and torsion, which is suggested in the collections of problems in differential geometry. Obviously, the question of the existence and form of the dependence between curvature and torsion of the curve belonging to the surface of Euclidean space other than a sphere is an example of a research problem. It was solved for a conical surface in (Stegantsev & Grechneva, 2019).

CONCLUSIONS AND PERSPECTIVES FOR FURTHER RESEARCH

The research activity should not be considered as a separate element of the educational process, but should become a natural part of learning. The paper demonstrates the possibilities of organizing students' research activities in teaching mathematical courses, using the theory of solving research problems, analyzes some possibilities of creating research problems, and considers methods of solving the proposed research problems (the method of the focal object, accumulation of educational material, research discussions). Mathematical courses are a favorable basis for ensuring research activities of university students. The article focuses on the inclusion of research tasks in the learning process on a regular basis. University teachers should improve their own experience in organizing students' research activities, as well as the experience of teachers from foreign universities who have been working on the implementation of research-based learning for some time. It is clear that the transition to this technology will be long and will require gradual changes in the curricula of the specialties. It will also require research into methods of increasing student motivation and self-esteem to ensure the effectiveness of research activities.

ACKNOWLEDGMENT

The authors thank anonymous reviewers for careful reading of the manuscript and helpful comments.

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| *Матеріал надійшов до редакції: 27.02.2025 р.* | *Прийнято до друку: 05.04.2025 р.* | *Опубліковано: 27.06.2025 р.* |